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Design of an Adaptive Backstepping Controller for a Two-Degree-of-Freedom Robot with a Simplified Control Law

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Abstract

This paper introduces a novel approach to desigin an adaptive backstepping controller for a specific class of nonlinear systems characterized by an affine structure in the input. The primary challenge in conventional backstepping methods lies in the high complexity of the control law and its heavy dependence on the derivatives of Lyapunov functions, which complicates practical implementation. This research proposes a simplified control law by eliminating redundant nonlinear feedback terms while maintaining asymptotic stability. The proposed method is implemented on a two-degree-of-freedom (2-DOF) robot and evaluated under two scenarios: an ideal environment and one subject to bounded disturbances. Simulation results demonstrate significant improvements in transient response, reduced oscillations, and faster convergence of tracking errors to zero compared to traditional backstepping techniques.

Keywords: Simplified backstepping control, Adaptive backstepping, 2-DOF robot, Lyapunov stability, Nonlinear cascaded systems.

1. Introduction

The precise control and stability of nonlinear systems have become a cornerstone of modern control engineering due to their extensive applications in strategic industries such as robotics [1,2], aerospace [3], and power systems [4]. Robotic systems, in particular, present a formidable challenge for designers due to their inherent nonlinearities, dynamic coupling, and parametric uncertainties.

While backstepping control offers high flexibility for cascaded systems by providing a systematic recursive design tool [5], its strict reliance on Lyapunov's second method often results in overly complex control laws. These laws contain numerous nonlinear terms that may not contribute significantly to performance but increase the computational burden on real-time digital processors. This complexity can lead to undesirable time responses or numerical instabilities during implementation.

The core contribution of this paper is the presentation of a new structure within the backstepping framework. By deliberately simplifying the control law and removing certain inherent terms from the traditional method, we maintain asymptotic stability while enhancing the system's dynamic performance. A key advantage of this approach is its independence from a specific choice of Lyapunov function, providing the designer with greater freedom and resulting in smoother responses in the presence of uncertainties.

2. Methodology and Controller Design

To elucidate the proposed method, consider the class of nonlinear systems defined by the following state space equations:

$$\dot{x} = f(x) + G(x)\,\xi\,, x(0) = x_0, \tag{1}$$
$$\dot{\xi} = f_a(x,\xi) + G_a(x,\xi)u, \xi(0) = \xi_0. \tag{2}$$

In the conventional design, using a virtual control and a sub-Lyapunov function, the final control law is derived to ensure that the derivative of the total Lyapunov function remains negative definite. The traditional control law is expressed as:

$$\begin{aligned} u = {G_a}^{-1}(x,\xi)\Big\{ &\frac{\partial^T \phi}{\partial x}[f(x) + G(x)\xi] \\ &- \frac{1}{2}P^{-1}G^T(x)\frac{\partial V_{sub}}{\partial x}(x) \\ &- f_a(x,\xi) \\ &- k\big(\xi - \phi(x)\big)\Big\}, k > 0. \end{aligned} \tag{3}$$

In contrast, this study proposes a simplified control law. The distinction lies in the removal of unnecessary nonlinear feedback components. The proposed control law is defined as follows:

$$u = G_a^{-1}(x,\xi)\left\{\frac{\partial^T \phi}{\partial x}[f(x) + G(x)\xi] - f_a(x,\xi) - k\big(\xi - \phi(x)\big)\right\}, k > 0. \quad (4)$$

3. Stability Analysis

The asymptotic stability of the closed-loop system under the proposed simplified control law is established in Theorem 1. For this purpose, a candidate Lyapunov function is utilized:

$$V(x,\xi) = V_{sub}(x) + \big(\xi - \phi(x)\big)^T P\big(\xi - \phi(x)\big). \quad (5)$$

The proof process involves partitioning the state space into two distinct regions: one where the derivative of the Lyapunov function is negative and another where it may temporarily be positive. By leveraging Class K and KL functions and applying specific lemmas (referencing Lemmas 1 to 4 in the manuscript), it is demonstrated that even in regions with a positive derivative, the boundedness of the states is guaranteed. Ultimately, the entire system converges to the origin. This mathematical analysis proves that simplifying the control law does not compromise the overall stability of the system.

4. Implementation on a 2-DOF Robot

To validate the theoretical findings, a two-link robot was chosen as the case study. The structure is illustrated in the following figure:

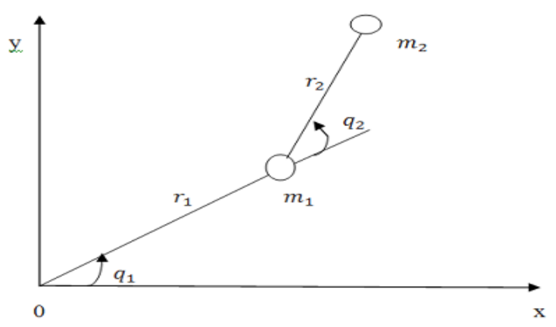

Figure 1. Two-link robot

The dynamics of this robot include the inertia matrix, Coriolis and centrifugal forces, and gravitational effects, modeled in a state-space format. With the state vector comprising joint positions and velocities, the dynamic equations are transformed into the standard form:

$$M(q)\ddot{q} + C(q,\dot{q}) + G(q) = \tau, \quad (6)$$

$$\dot{X}_1 = X_2,$$
$$\dot{X}_2 = f(X_1, X_2) + g(X_1)U. \quad (7)$$

Subsequently, the adaptation law for the unknown parameters was developed based on the backstepping mechanism. This ensures that the controller can maintain tracking performance despite parametric variations and uncertainties.

5. Results and Discussion

Simulations were conducted under two primary scenarios. In the first scenario, tracking performance was evaluated under ideal conditions. The results indicate rapid convergence of the state variables to the desired trajectory:

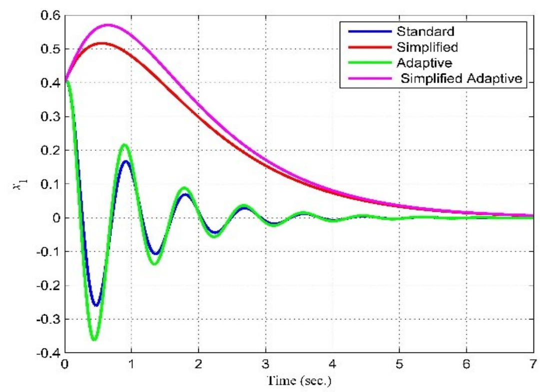

Figure 2. The state trajectory of x_1

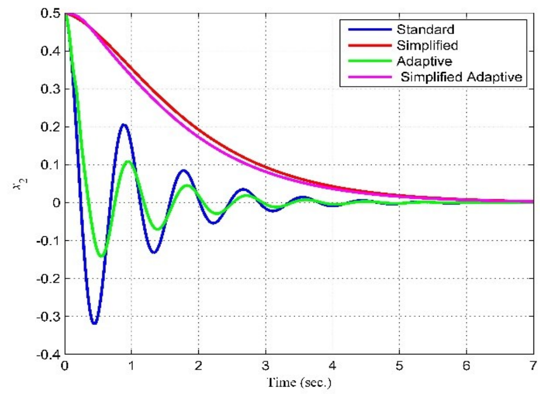

Figure 3. The state trajectory of x_2

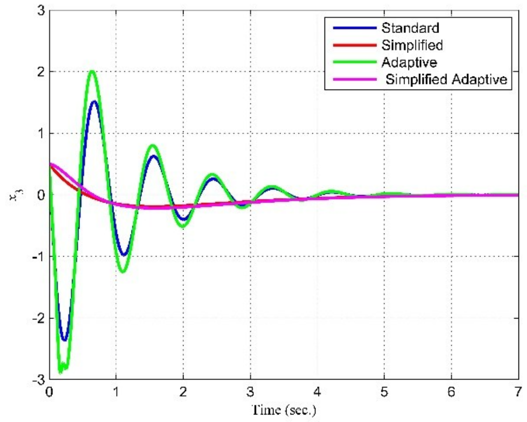

Figure 4. The state trajectory of x_3

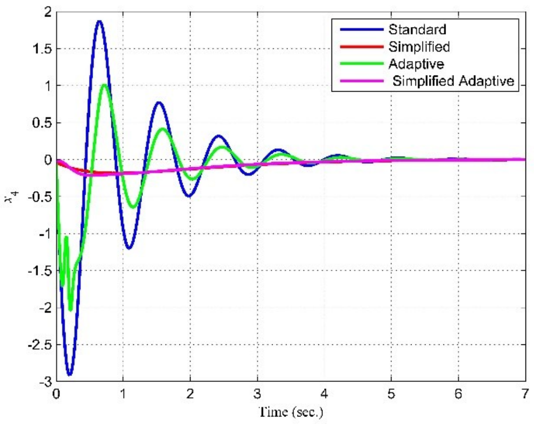

Figure 5. The state trajectory of x_4

Furthermore, the phase plane analysis reveals a smoother behavior with minimal oscillations in the proposed method compared to the conventional approach:

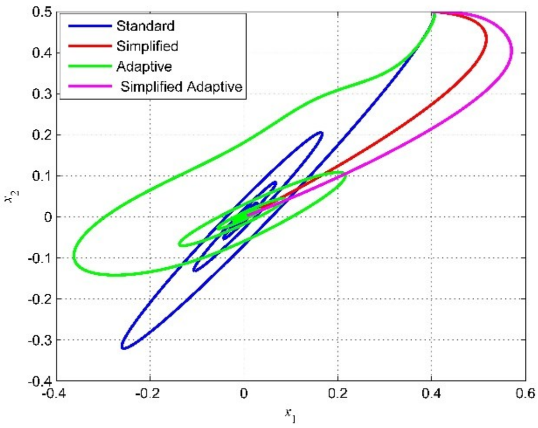

Figure 6. Phase plane of $x_1 - x_2$

In the second scenario, the system's robustness against external disturbances was tested. Upon applying bounded random disturbances, the "Adaptive Backstepping with Simplified Law" demonstrated superior stability, exhibiting lower overshoot and undershoot than traditional methods.

The control signals also highlight a reduced computational load and more refined actuator behavior in the simplified method:

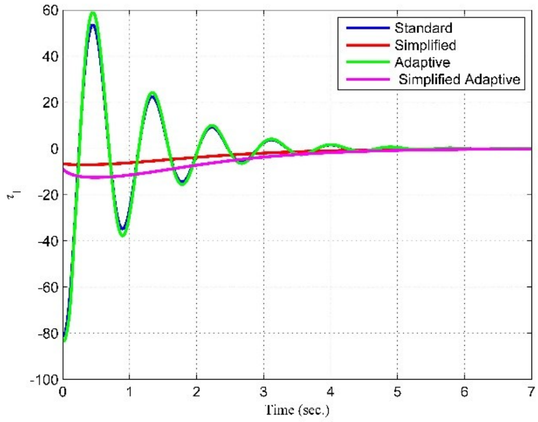

Figure 7. Control law τ_1 for link 1

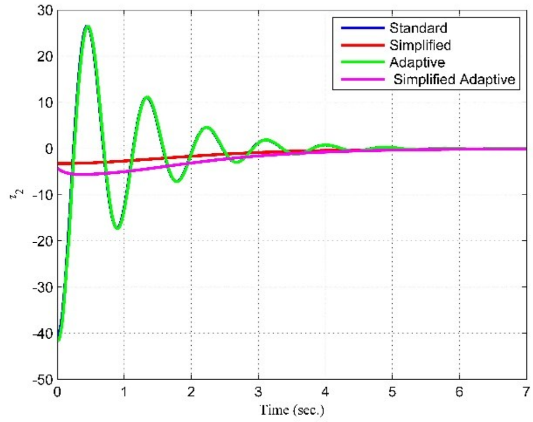

Figure 8. Control law τ_2 for link 2

A quantitative comparison based on performance indices such as RMS error and settling time is provided in the tables below, confirming the definitive advantages of the proposed approach:

Table 1. Comparison of performance indices of different control methods under disturbance

Standard Backstepping Controller				
Variable	Overshoot	Undershoot	Settling Time	RMS Error
x_1	0.1832	0.0033	3.3450	0.1599
x_2	0	0.0022	2.5850	0.4187
x_3	0.0098	0.1763	3.6150	0.0623
x_4	0.0216	0.8407	3.7550	0.2368
τ_1	0.0199	13.3438	3.9750	4.5052
τ_2	0.0261	7.7504	3.2950	3.2486
Backstepping Controller with Simplified Control Law				
Variable	Overshoot	Undershoot	Settling Time	RMS Error
x_1	0	0.0004	1.4600	0.0384
x_2	0	0.0007	1.6650	0.0617
x_3	0.0050	0.1568	2.3200	0.1356
x_4	0.0109	0.2788	7.8500	0.2386

τ_1	0.3387	0.0113	0.0850	0.6862
τ_2	0.1394	0.0142	1.2600	0.3607
Conventional Adaptive Controller				
Variable	Overshoot	Undershoot	Settling Time	RMS Error
x_1	0.2180	0.0020	2.3300	0.1032
x_2	0	0.0228	1.4800	0.1978
x_3	0.0075	0.4018	4.3700	0.1123
x_4	0.0618	0.3899	3.7400	0.2181
τ_1	0.0329	12.8347	2.1350	4.8934
τ_2	0.0270	6.4516	1.6900	2.6253
Adaptive Controller with Simplified Control Law				
Variable	Overshoot	Undershoot	Settling Time	RMS Error
x_1	0	0.0004	1.3300	0.0384
x_2	0	0.0006	2.2450	0.0603
x_3	0.0042	0.1496	5.4550	0.1391
x_4	0.0134	0.2786	5.4550	0.2406
τ_1	0.0111	0.0787	1.0850	1.0930
τ_2	0.0114	0.0577	: 0.9250	0.5638

6. Conclusions

This paper introduced a "Simplified Control Law" framework within the backstepping design process. The most significant finding is that removing complex nonlinear feedback terms does not jeopardize stability; rather, it eliminates unnecessary computational noise and interference, leading to a higher-quality time response. The independence of the control law from the Lyapunov function choice significantly increases the designer's flexibility in managing complex robotic systems. For future work, implementing this algorithm on industrial robotic arms while considering actuator dynamics is highly recommended.

7. References

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