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High-Order Analysis of Vibration Energy Harvesting from Sandwich Beams with Composite Face Sheets

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Abstract

This study analyzes the vibration response of a cantilever sandwich beam equipped with a piezoelectric layer as an energy harvester. The main objective is to assess how composite layup arrangement, variations in geometric/mechanical parameters, and modeling assumptions affect the harvested power and the overall efficiency. The sandwich beam is modeled using higher-order shear deformation theories and the finite element method. Effects of Poisson's ratio in the composite face sheets are incorporated, and the influence of through-thickness core strain as well as shear locking is investigated in detail. Two composite configurations are considered: fully covered and partially covered face sheets. Numerical results demonstrate that adding composite face sheets increases the output voltage and enhances the harvester performance. Moreover, the higher-order model provides improved accuracy in capturing the effects of core thickness and fiber orientation. Parametric studies—including core thickness, concentrated proof-mass length, and fiber direction—show that these factors play a significant role in tuning natural frequencies and improving energy harvesting efficiency. The findings highlight the importance of employing accurate, shear-locking–resistant models for the design of sandwich vibration harvesters, particularly for relatively large core thicknesses.

Keywords: Energy harvesting, Sandwich beam, Composite surface, Higher-order theory, Through thickness strain, Shear locking

1. Introduction

The increasing adoption of smart structures and wireless sensing networks has highlighted the need for reliable and sustainable power sources for low-power electronic devices. Conventional chemical batteries suffer from limited lifespan, maintenance costs, and environmental concerns, motivating researchers to explore energy harvesting from ambient vibrations as an eco-friendly alternative [1, 2]. Among various transduction mechanisms, piezoelectric energy harvesting has attracted significant attention due to its high power density and capability to convert mechanical strain into electrical energy without external power requirements. Nevertheless, most conventional piezoelectric harvesters exhibit efficient operation only near their resonance frequency, while many mechanical systems experience broadband, low-frequency vibrations (10–200 Hz) [3]. Therefore, enhancing low-frequency performance and widening the operational bandwidth remain major challenges.

To address these limitations, sandwich-type beam configurations have been widely studied for improving the electromechanical response of piezoelectric harvesters. Zhang Yang et al. [3] conducted both analytical and experimental investigations on piezoelectric sandwich harvesters and demonstrated that employing a sandwich substrate—rather than a traditional monolithic base—can reduce resonance frequency and increase output voltage through appropriate adjustment of geometric and material parameters. In a related contribution, Dao et al. [4] derived a closed-form analytical solution for a sandwich beam with piezoelectric layers under dynamic loading, incorporating higher-order shear deformation effects. Their results emphasized the strong influence of geometric configuration and excitation characteristics on harvested voltage and overall efficiency.

The novelty and contribution of the present work lie in the development of a refined, high-order shear deformation theory-based finite element model for clamped composite sandwich beams integrated with piezoelectric layers. This model systematically incorporates the effects of through-thickness shear and normal strains within the core, the influence of Poisson's ratio for both face sheets and the core, and employs a field-consistent formulation (FCR) to effectively suppress shear locking. The study also investigates the electromechanical response under both full-coverage and localized piezoelectric patch configurations, providing a more realistic and accurate assessment of energy harvesting potential. By considering these critical factors, this research aims to offer a more reliable tool for the design and optimization of efficient composite sandwich beam energy harvesters for various low-power applications.

2. Methodology

The geometry of configurations A and B, including piezoelectric patches and a tip mass, is shown in Figure 1.

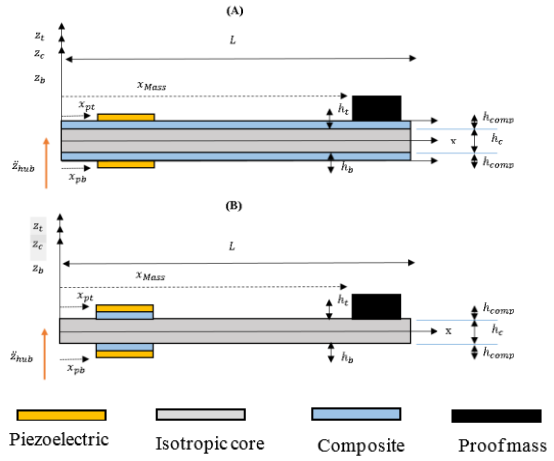

Figure 1.Configuration (A) and (B)

Kinematic relations

The core displacement field is represented using a higher-order shear deformation theory ([5]), where axial (u) and transverse (w) displacements vary as third- and second-order polynomials through the thickness (Eq. (**1**)). The face sheets follow Euler–Bernoulli assumptions (Eq. (**2**)). By solving the four algebraic equations obtained from enforcing displacement continuity at the interfaces between the face sheets and the core, the displacement variables w_1, u_3, u_2 and w_2 are expressed in terms of the remaining displacement variables. Consequently, seven independent displacement variables remain, namely u_0‘ u_1‘ w_0‘ u_{t0}‘ u_{b0}‘ w_{t0}, and w_{b0}. The strain–displacement relations are obtained from linear kinematics (Eq. (**3**)), with detailed expressions for the core and face sheets derived from the same framework.

$$u = u_0 + zu_1 + z^2u_2 + z^3u_3$$
$$w = w_0 + zw_1 + z^2w_2 \tag{1}$$

$$u_m = u_{m0} - z_m\frac{\partial w_m}{\partial x_m}, w_m = w_{m0}, \quad m: t, b \tag{2}$$

$$\varepsilon_{xx} = \frac{\partial u}{\partial x}, \varepsilon_{zz} = \frac{\partial w}{\partial z} \tag{3}$$

$$\varepsilon_{xz} = \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)$$

Constitutive relations

For the core, applying the transverse normal stress condition $\sigma_{yy} = 0$ to the Hooke constitutive relations [6] leads to the simplified constitutive equation given in (Eq. (**4**)) and the corresponding stress resultants (Eq. (**5**)).

$$\sigma_{xx} = \frac{E}{(1-\nu^2)}[(\varepsilon_{xx} + \nu\varepsilon_{zz}],$$
$$\sigma_{zz} = \frac{E}{(1-\nu^2)}[(\varepsilon_{xx}\nu + \varepsilon_{zz}], \tag{4}$$
$$\sigma_{xz} = G\varepsilon_{xz}$$

$$N^i_{\alpha\beta} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_{\alpha\beta} z^i\, dz, i = 0,1,2,3\,, \qquad \alpha, \beta: x, z \tag{5}$$

For the composite–piezoelectric face sheets, the electromechanical constitutive relation is introduced using the standard piezoelectric stress–charge form. Assuming a uniform electric field through thickness, the stress components reduce to three dominant terms. After coordinate transformation to the global (xy) system, the laminate stress-resultant relations follow the classical A, B, and D matrices (Eq. (**6**)). The electric displacement resultant is similarly obtained (Eq. (**7**)).

$$\begin{Bmatrix} N \\ M \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix}\begin{Bmatrix} S_0 \\ \kappa \end{Bmatrix} + \sum_{k=1}^{n} \int_{z_{k-1}}^{z_k} \begin{bmatrix} I_3 \\ zI_3 \end{bmatrix} [R_T]_k^{-1} \begin{Bmatrix} e_{31} \\ e_{32} \\ 0 \end{Bmatrix} \frac{\phi_k}{h_k} dz \tag{6}$$

$$D_k = \{e_{31} \quad e_{32} \quad 0\}_k [R_S]_k \{S\}^{(xy)} - \varepsilon_k \frac{\phi_k}{h_k}$$
$$= \{e_{31} \quad e_{32} \quad 0\}_k [R_S]_k [I_3 \quad zI_3] \begin{Bmatrix} S_0 \\ \kappa \end{Bmatrix} - \varepsilon_k \frac{\phi_k}{h_k} \tag{7}$$

Where e_{31} and e_{32} are the piezoelectric coefficients, I is the identity matrix, and ϕ_k is the electric potential.

Poisson’s ratio effect

In the face sheets, N_{yy}‘ N_{xy}‘ M_{yy}‘ M_{xy} are zero, and only N_{xx} and M_{xx} are non-zero. By expressing the strains and curvatures in the y-direction in terms of those in the x-direction and substituting them into N_{xx} and M_{xx} we obtain:

$$N_{xx} = (\bar{C}_1\epsilon^0_{xx} + \bar{C}_2\kappa_{xx}) + F_1$$
$$M_{xx} = (\bar{C}_3\epsilon^0_{xx} + \bar{C}_4\kappa_{xx}) + F_2 \tag{8}$$

$$D_3 = \bar{G}_1 \epsilon_{xx}^0 + \bar{G}_2 \kappa_{xx} + \bar{G}_3 E_3 \tag{9}$$

where $\bar{C}_1$,$\bar{C}_2$,$\bar{C}_3$,$\bar{C}_4$,$\bar{G}_1$,$\bar{G}_2$,$\bar{G}_3$,$F_1$and F_2 are constants depending on the A, B, and C matrices and the electrical properties of the piezoelectric material.

Energy formulation and finite element model

Strain energies of the core and face sheets (Eq. (**10**)), internal electrical energy (Eq. (**11**)), and kinetic energy of the structure including the tip mass are formulated within the Lagrangian framework.

$$U = \frac{1}{2}\int \varepsilon_{ij}\,\sigma_{ij}\,dV \tag{10}$$

$$W_{ie} = \frac{1}{2}\int_{V_p} \mathbf{E}^t \mathbf{D} dV_p \tag{11}$$

A three-node finite element with nine degrees of freedom per node is employed. Axial variables use Lagrangian shape functions, whereas transverse displacements use Hermite interpolation.

Shear-locking treatment and governing equations

To avoid shear locking in the high-order model, the displacement associated with the shear term is modified using a field-consistency redistribution approach (Eq. (**12**)).

$$S_1^{(1)} = \frac{1}{6} - \frac{\xi}{2}, S_2^{(1)} = \frac{2}{3}, S_3^{(1)} = \frac{1}{6} + \frac{\xi}{2} \tag{12}$$

where ξ is the dimensionless coordinate of each element, and $S_i^{(1)}$ is the Lagrangian shape function.

Substitution of the displacement field into the energy relations yields the element stiffness, mass, electromechanical coupling, and capacitance matrices. After assembling all elements and applying boundary conditions, the coupled electromechanical governing equations are obtained. Under harmonic excitation, the steady-state voltage output is extracted from the final frequency-domain form of the governing equations.

3. Discussion and Results

Figure 2 shows the effect of neglecting Poisson's ratio on the output voltage of the sandwich beam with composite face sheets and piezoelectric layers, evaluated at the resonance frequency for each core thickness. For both 0° and 90°fiber orientations, noticeable differences appear between the results with and without Poisson coupling, indicating that neglecting Poisson's ratio can lead to significant errors in predicting the electromechanical response.

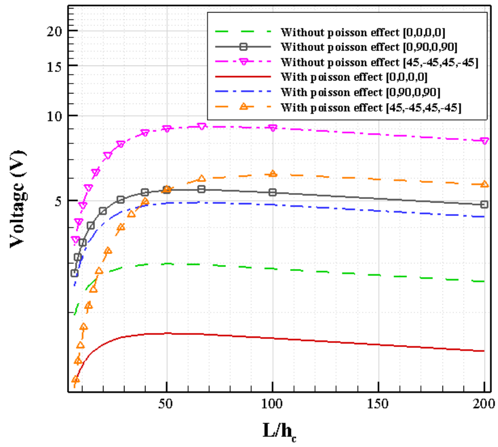

Figure 2. Effect of Poisson's ratio on the voltage versus core thickness (excitation amplitude: 1 g).

Figure 3 and Figure 4 show the effect of piezoelectric patch length in configurations A and B for a single composite face sheet without added mass, using a 1 mm core and 0.2 g base excitation. In Configuration A (Figure 3), all fiber orientations yield maximum voltage at shorter patch lengths, with output decreasing as the patch becomes longer. The highest potential, about 21 V at the 90° orientation, exceeds the non-composite case (18 V), indicating that adding a composite face sheet enhances energy harvesting efficiency.

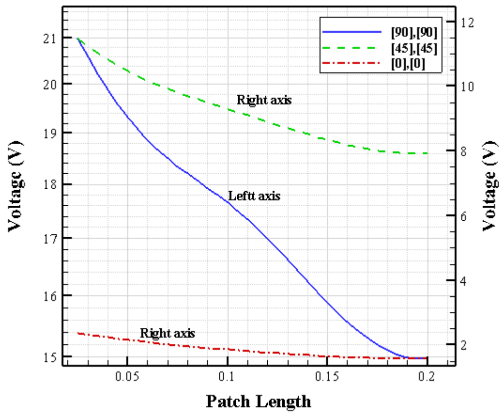

Figure 3. Effect of the piezoelectric patch length for configuration (A).

Figure 4 illustrates Configuration B (segmented layers), where maximum voltage occurs at shorter patch lengths. Unlike Configuration A, the voltage trend for 0° and 90° fiber orientations is non-monotonic, leveling off or slightly increasing with length. The peak output (19.5 V at 90°) exceeds the non-composite case (18V), confirming that composite integration enhances energy harvesting performance.

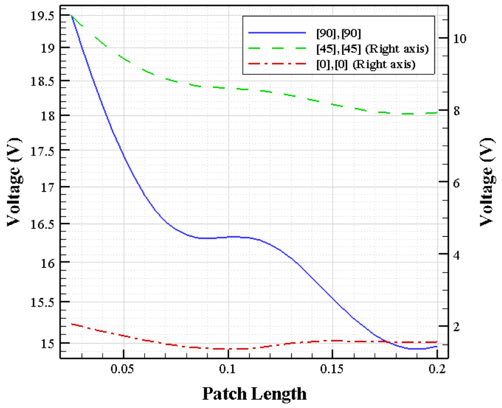

Figure 4. Effect of the piezoelectric patch length for configuration (B).

Figure 5 shows the effect of including transverse normal (through-thickness) strain in the core for fiber orientations of 90° and 45° in Configuration A with localized piezoelectric coverage, under excitation at the resonance frequency of each core thickness. Neglecting this strain underestimates the generated voltage for both orientations. The maximum error (≈9.26%) occurs at L/h=12, where both shear and through-thickness deformations are significant. As the core becomes thicker and shear deformation dominates, the error decreases, while for thinner cores it falls below 3%. At lower thickness ratios, the error for the 45°orientation is slightly higher than for90°.

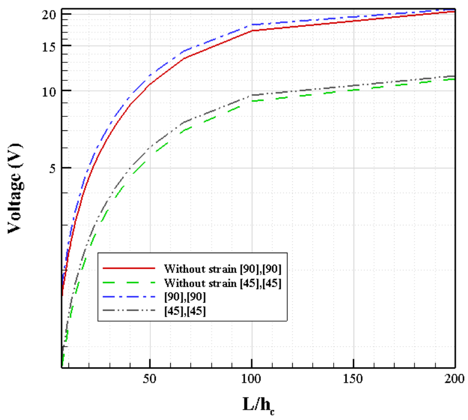

Figure 5.Effect of considering the transverse-normal strain in the core on the output voltage.

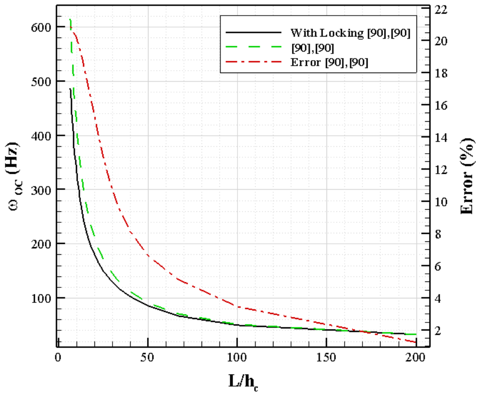

Figure 6. Effect of neglecting shear locking on the open-circuit frequency ([90] and [90]).

Figure 6 illustrates the influence of shear locking on the open-circuit natural frequency for the 90° fiber orientation. As core thickness increases, the locking-affected model predicts lower frequencies than the locking-free formulation. The error peaks near 20% at low L/h ratios and falls below 2% for slender beams. This high sensitivity at 90° arises from reduced bending stiffness, where transverse fiber orientation amplifies shear effects. Consequently, a shear-locking-free formulation is essential for thick sandwich beams and configurations with low flexural rigidity.

4. Conclusions

The study examined the electromechanical behavior of a cantilevered sandwich beam featuring composite face sheets and a piezoelectric layer, with the primary goal of assessing how composite-layer placement and key geometric parameters influence the harvested electrical output. The motivation lies in enabling adaptable energy harvesters whose performance can be tuned for various operational environments. To this end, the core was modeled using a higher-order shear deformation theory including transverse-normal strain, while the face sheets followed Euler–Bernoulli assumptions with explicit incorporation of Poisson's ratio effects. A three-node finite element with Hermitian–Lagrangian interpolation and shear-locking-free formulation was employed.

Two composite-layer configurations were investigated: full-length face-sheet coverage and partial coverage restricted to the piezoelectric patch region. Results demonstrated that Poisson's ratio plays a notable role in predicting the electromechanical fields, particularly when composite layers are present. Full-length composite coverage increased the peak harvested voltage from 18 V to nearly 21 V. The higher-order formulation enabled accurate sensitivity analysis with respect to core thickness, showing that mass-length ratio, fiber orientation, and beam geometry allow tuning of the natural frequency from approximately 18 Hz up to 90 Hz depending on design parameters.

The influence of transverse‑normal strain in the core was also quantified. Omitting this strain led to underprediction of voltage, with maximum error reaching about 9% for large core thicknesses (L/h = 12), though the error dropped below 3% for thinner cores. Near resonance, the discrepancy became significant for fiber orientations of 45° and 90°, emphasizing the importance of including this kinematic effect.

Shear‑locking behavior was assessed by comparing locking‑free and locking‑affected models. For thick cores, the locking‑affected formulation underestimated natural frequencies and overestimated voltage, with errors up to 14% for 0° fiber angle and nearly 20% for 90°. At high L/h ratios, the error decreased to negligible levels for 0° and below 2% for 90°, reinforcing the necessity of locking‑resistant finite‑element formulations.

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