

## The Effect of Applying Axial Tensile Force to the Workpiece on Vibration Reduction in the Face milling Process

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### Abstract

Vibrations in the machining and turning process of engineering parts are one of the common problems in the industry that have a significant impact on the surface quality of machined parts. In this paper, the effect of applying axial force to the part during the turning process is investigated as a solution to reduce the effect of the vibration phenomenon. For this purpose, the part is modeled as a clamped-hinged beam and the turning tool is modeled as a moving harmonic load. According to Rayleigh theory, the equation of motion for the beam is derived. Then, using the Galerkin method, the equation of motion is solved and the natural frequencies and dynamic response of the beam are obtained. To validate the results, the deflection at the midpoint of the beam in the present model is compared with the results from the COMSOL software. The validation results indicate good agreement between the numerical model and the COMSOL model. The results show that the axial force, amplitude, and speed of the harmonic force have a great effect on the natural frequencies, maximum deflection, and frequency responses of the beam. Furthermore, as the axial force increases, the maximum deflection of the beam decreases and its natural frequencies increase. These changes indicate the positive effect of applying the axial force in reducing the vibration of the beam.

**Keywords:** vibration in the Face milling process, Fixed-simply supported beam, moving harmonic load, axial force.

### 1. Introduction

Vibrations in machining processes are one of the main factors reducing the surface quality of parts and productivity. These vibrations are divided into two general types: forced and self-excited (chatter). Forced vibration is caused by external excitations such as machine vibration, cutting forces, and workpiece surface roughness [1]. Self-excited vibration occurs in two cases: when the length-to-thickness ratio of the workpiece or the face milling tool is large [2, 3]. Both types of vibration cause surface deviation and a drop in part quality. Therefore, the study of dynamic characteristics and improving machining quality has been the focus of many research works. In most studies, the workpiece has been modeled as a clamped-hinged beam and the face milling tool as a moving load.

All abstracts should be written in English. Subtitles of the extended abstract should be composed of the sections of abstract and keywords, introduction and objective, method, results and discussion and conclusion. Extended abstracts can contain figures, tables, formulations or images. Lee et al. [4] investigated the effect of workpiece thickness and the tailstock axial force on the natural frequency using Euler–Bernoulli beam theory. Liu et al. [5] modeled the

workpiece as a cantilever beam and the face milling force as a simple moving load, and validated the results with experimental results. Furthermore, the effect of applying a moving load on the transverse vibration of beams has been investigated in several studies. Onda et al. [6] investigated the free vibration of an Euler–Bernoulli beam under an axial force. Their results showed that increasing the axial force increases the natural frequencies.

In this paper, the effect of applying an axial force as a solution to reduce the vibration of the workpiece during the face milling process is investigated. The workpiece is modeled as a clamped-hinged beam under an axial force, and the face milling tool is modeled as a moving harmonic load. The governing equations are derived by considering rotary inertia and are solved using the Galerkin method. The results show that applying the axial force reduces the maximum displacement and increases the natural frequencies, which indicates the positive effect of applying the axial force in reducing vibrations in the face milling process. Another innovation of this research is the simultaneous investigation of the effect of the axial force and the moving harmonic load on the vibrational behavior of a clamped-hinged beam.

## 2. Modeling

In the modeling, considering forced vibration, the face milling tool is modeled as a moving harmonic force and the beam as a clamped-hinged beam. To investigate the effect of the axial force, a tensile force ( $P$ ) is applied at the end of the beam. The beam of length  $L$  is modeled using Rayleigh theory, which provides higher accuracy compared to the Euler–Bernoulli theory due to taking rotary inertia into account. The beam dimensions include thickness  $h$  and width  $b$ . The equation of motion based on Rayleigh theory is presented in Equation (1), the moving harmonic force in Equation (2), and the boundary conditions in Equation (3) [7]. To solve Equation (1), the Galerkin method is used.

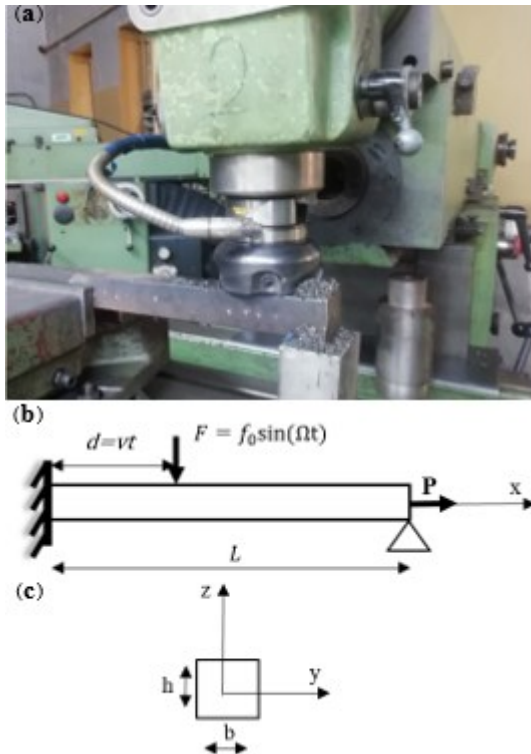


Figure 1. a) a cantilever beam subjected to face milling with the FP4MH milling machine in industry, b) the side view of the cantilever beam model under a moving load and axial force, and c) side view.

## 3. Results and Discussion

To validate the model and the solution of the equations, the deflection variations at the midpoint of the beam were compared with the results from the COMSOL software modeling, which showed good agreement (Figure 2). For modeling in COMSOL, after mesh sensitivity analysis, 24 elements with a size of 0.08 were selected.

The effect of changes in the vibration frequency of the face milling tool on the maximum dynamic deflection versus face milling speed is shown in Figure 3. Based on Figure 3, increasing the face milling speed significantly reduces the maximum dynamic deflection

only at frequencies near resonance, which indicates the importance of reducing tool vibration. Furthermore, Table 1 shows that the axial force increases the natural frequencies of the beam.

$$EI \frac{\partial^4 w(x,t)}{\partial x^4} + \rho A \frac{\partial^2 w(x,t)}{\partial t^2} - \rho I \frac{\partial^4 w(x,t)}{\partial x^2 \partial t^2} - \quad (1)$$

$$P \frac{\partial^2 w}{\partial x^2} = f(x,t)$$

$$f(x,t) = f_0 \sin(\omega t) \delta(x - v_0 t) \quad (2)$$

$$\begin{aligned} w(0,t) = 0 & \quad EI \frac{\partial^2 w(x,t)}{\partial x^2} \Big|_{x=l} = 0 \\ \frac{\partial w(x,t)}{\partial x} \Big|_{x=0} = 0 & \quad w(l,t) = 0 \end{aligned} \quad (3)$$

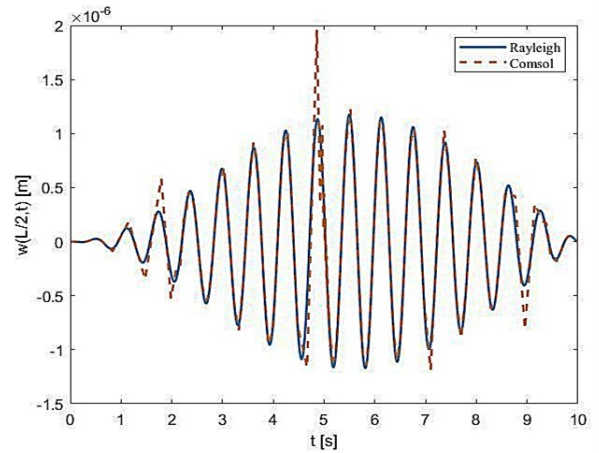


Figure 2. Comparison of the results of the present model and COMSOL software

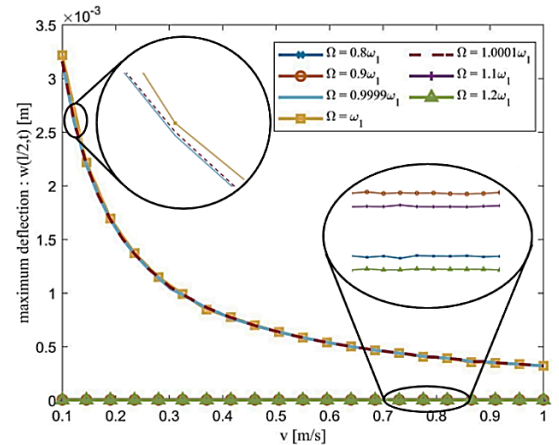


Figure 3. Effect of axial force on the maximum dynamic deflection versus face milling speed (moving load)

Table 1. The natural frequencies of the beam for different values of the axial force

| Axial force [N] | First natural frequency | Third natural frequency | Fifth natural frequency |
|-----------------|-------------------------|-------------------------|-------------------------|
| 0               | 977.47019               | 6553.04617              | 16824.6089              |
| 5000            | 978.96397               | 6554.83666              | 16826.4435              |
| 10000           | 980.45531               | 6556.62666              | 16828.2780              |

Figure 4 shows the maximum deflection of the beam versus the feed speed of the face milling tool for different values of the axial force. As can be seen, the maximum deflection-speed curves have increasing and decreasing regions at specific speeds, reaching a peak at a certain speed and then decreasing, which is called the critical speed [8, 9]. According to Figure 4, as the axial force increases, the maximum deflection of the beam decreases and the critical speed increases, respectively. Figures 5 and 6 show the effect of the face milling force magnitude and speed, respectively, on the maximum deflection of the beam at the resonance frequency. As observed, with increasing face milling force and decreasing face milling speed, the maximum deflection increases. According to Figure 7, which shows the effect of the axial force on the resonance frequency amplitude, increasing the axial force reduces the deflection amplitude of the beam at the resonance frequency. It should be noted that, due to neglecting the damping effects in the equations, in Figures 4 to 7, the vibration amplitude at the resonance frequency has a

finite value. The reason for the finite vibration amplitude in the absence of damping is the limited time duration and the transient nature of the moving force.

#### 4. Conclusions

In this paper, the effect of the axial force on reducing the vibration of the beam under face milling process has been investigated. The workpiece was modeled as a clamped-hinged beam under an axial force, and the face milling tool was modeled as a moving harmonic load, then the governing equations were derived. The natural frequencies and dynamic response were calculated using a numerical method. Validation through simulation in COMSOL software confirmed good agreement of the results. The results showed that increasing the axial force reduces the dynamic deflection and resonance amplitude, and increases the critical speed and natural frequencies. Based on the obtained results, it can be concluded that applying an axial force in face milling process can reduce the deflection and the resonance frequency amplitude.

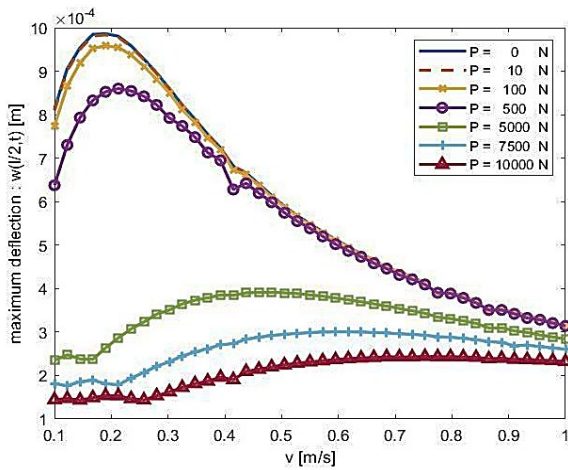


Figure 4. Effect of axial force on the maximum dynamic deflection versus face milling speed (moving load)

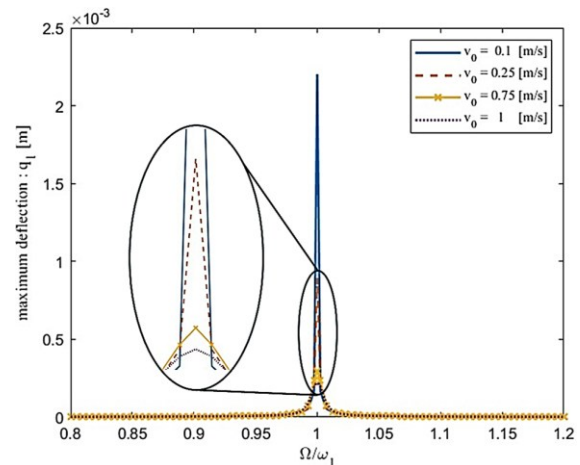


Figure 6. Effect of face milling speed on the frequency response of the beam

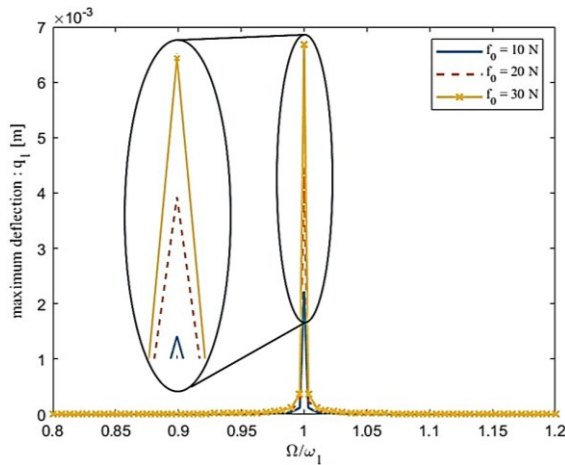


Figure 5. Effect of the face milling force magnitude on the frequency response of the beam

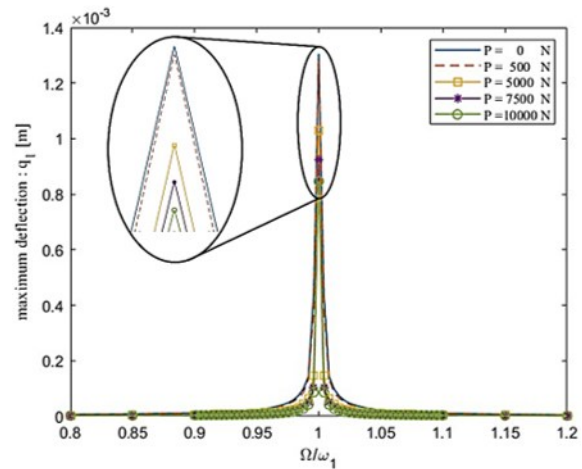


Figure 7. Effect of axial force on the frequency response of the beam

## 5. References

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