

## Modeling of the critical force in three-dimensional manipulation using the design of experiments method

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### Abstract

In this study, the modeling of the critical manipulation force of particles in three-dimensional space was investigated using the Design of Experiments (DoE) approach. The main objective was to analyze the influence of geometric and control parameters on the critical force required for particle manipulation using an Atomic Force Microscope (AFM). Five key parameters including particle radius ( $R_p$ ), cantilever thickness ( $T$ ), cantilever length ( $L$ ), cantilever width ( $W$ ) and tip height ( $H$ ) were selected as input variables, while the critical forces in the X and Y directions were modeled and analyzed as output responses. Based on the results from 27 designed experiments and the statistical analysis of the derived regression model, it was found that cantilever thickness and particle radius had the most significant impact on increasing the critical manipulation force, whereas increasing the cantilever length and tip height led to a reduction in this force. The 3D surface plots revealed that the interaction effects between the parameters were considerable, and the overall manipulation performance strongly depended on the appropriate combination of geometric and physical properties. Additionally, residual analysis confirmed the normal distribution of errors and the high accuracy of the model ( $R^2 > 99\%$ ). The findings of this study can serve as a foundation for the optimal design and development of advanced manipulation tools.

**Keywords:** Three-dimensional nanomanipulation, critical force, design of experiments, regression model, and geometric parameters.

### 1. Introduction

Manipulation of particles at the micro- and nanoscale, particularly in three-dimensional space, is one of the fundamental challenges in fields such as nanotechnology, biotechnology, and precision material structuring. This process relies on the controlled displacement of extremely small particles using tools such as the Atomic Force Microscope (AFM), which provides high accuracy and stability in nanoscale environments. Despite significant advancements in this area, one of the critical issues remains the determination of the critical force required to initiate particle movement, i.e., the force needed to overcome surface adhesion, friction, and contact resistance, thereby ensuring successful manipulation.

A precise understanding of the interplay between influencing factors such as particle geometry, mechanical properties of the AFM cantilever, contact conditions, and substrate surface characteristics—is essential for predicting and controlling manipulation behavior. Parameters such as particle radius, cantilever thickness, length, width and tip height all play a significant role in determining the critical force. Therefore, systematic identification and analysis of these variables are crucial for optimizing

the manipulation process.

In recent studies, non-linear and non-classical models have been employed to accurately investigate the interaction forces between Atomic Force Microscopes (AFM) and carbon nanotubes. These works have analyzed the mechanical behavior of nanostructures using contact theories and Timoshenko beam theory. The results indicate that considering scale effects plays a critical role in improving modeling accuracy; the differences between classical and non-classical models can become significant under specific conditions.[1]

Atomic Force Microscopy (AFM) enables the determination of mechanical properties such as Young's modulus. The AFM-based nanoindentation technique has been effectively utilized to accurately determine the elastic modulus of samples with a wide range of stiffness, including live cells in their native environment. Key factors for the successful implementation of this method include proper probe selection, calibration, sample homogeneity, surface roughness, and statistical data analysis, all of which contribute to providing comprehensive information on the sample's composition and structure.[2]

In this study, the Design of Experiments (DoE) method is employed to model and perform sensitivity analysis of the critical manipulation force in 3D. The effects of six key parameters on the critical forces in

both the X and Y directions are investigated. The results provide a clear understanding of the relationship between the system's physical parameters and its mechanical response, offering a

robust foundation for the precise design and control of nanoscale manipulation in both industrial and research applications. The overall schematic of the process can be seen in Figure 1.

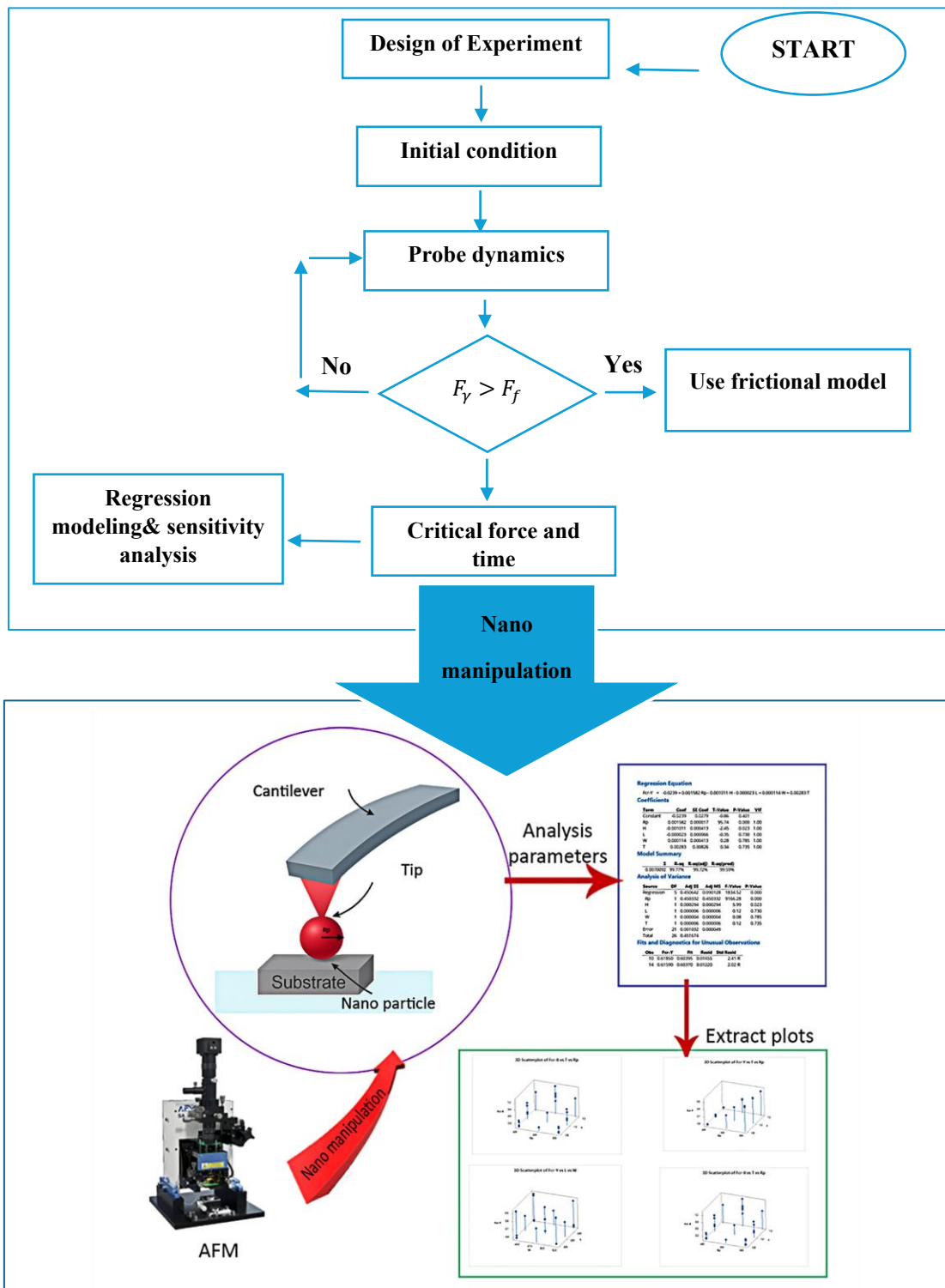


Figure 1. The overall schematic of the process

## 2. Methods and materials

To investigate the relationships between input parameters and output forces, multiple linear

regression analysis was performed separately for the critical manipulation forces in the X and Y directions. For the regression model of the critical force in the X direction, the coefficient of determination ( $R^2$ ) was

found to be 95.25%, indicating a good fit of the model. In this model, cantilever length (L) and thickness (T) were statistically significant ( $p < 0.05$ ), with cantilever thickness identified as the most influential parameter. The regression equation for the critical manipulation force in the X direction was obtained as follows:

$$F_{crX} = -0.044 + 0.000006 Rp - 0.00017 H - 0.006433 L + 0.00989 W + 1.6163 T \quad (1)$$

Subsequently, the regression model for the critical force in the Y direction demonstrated an even stronger fit, with a coefficient of determination ( $R^2$ ) of 99.77%.

According to the parameter coefficients in this model, the particle radius (Rp) and tip height (H) were statistically significant. The final regression equation for this model is as follows:

$$F_{crY} = -0.0239 + 0.001582 Rp - 0.001011 H - 0.000023 L + 0.000114 W + 0.00283 T \quad (2)$$

### 3. Results and Discussion

In this section, the interactive effects of various design parameters on the critical three-dimensional manipulation force of particles were investigated using 3D surface plots.

The responses include the critical manipulation force in the X and Y directions, derived based on combinations of different levels of the input variables. The results of this analysis are illustrated in Figure 2.

In diagram (a), as the cantilever thickness increases from 0.8 to 1.2  $\mu\text{m}$ , the critical manipulation force in the X direction rises significantly, particularly at higher particle radii.

This behavior indicates that cantilever thickness is one of the key parameters in controlling the applied forces during manipulation. A thicker cantilever increases resistance to deformation, thereby enhancing the force required in the X direction.

In diagram (b), the critical force in the Y direction also increases with cantilever thickness, although the rise is more gradual compared to the X direction.

Notably, for smaller particle radii, the difference between various thickness levels is less pronounced. This suggests that the cantilever thickness, in combination with particle size, plays a controlling role in the manipulation process.

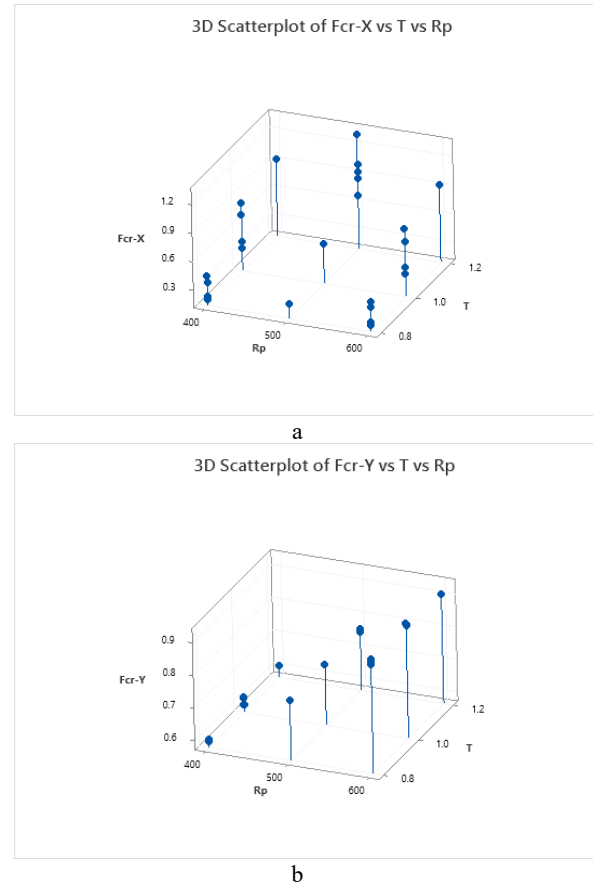


Figure 2. Diagram of the influence of parameters on critical force

### 4. Conclusions

In this study, the Design of Experiments (DoE) approach was employed to investigate the behavior of the critical manipulation force in three-dimensional space and to identify the key parameters influencing it. The main objective was to develop a mathematical model and conduct sensitivity analysis of five critical parameters, including particle radius (Rp), cantilever thickness (T), cantilever length (L), cantilever width (W) and tip height (H) on the critical forces in the X and Y directions. These variables were systematically varied across 27 designed experiments, and the resulting output data were analyzed using regression modeling and 3D response surface plots.

The results indicated that the particle radius and cantilever thickness had a direct and significant effect on increasing the critical manipulation force. In other words, as these two parameters increased, the required force to initiate particle manipulation rose markedly. This finding highlights the critical role of particle mechanical properties and the stiffness of the control system in ensuring stable manipulation. In contrast, increasing the cantilever length and tip height led to a decrease in the critical force value.

## 5. References

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