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Using Fractional Derivatives in the Analysis of Magneto Hydrodynamic Flow in a Diverging Channel with Concentrated Flow Injection from the Walls with Mutable Velocity

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Abstract

This study examines the influence of fractional derivatives on the issue of concentrated flow injection from the walls into a non-concentrated magneto hydrodynamic flow within diverging channels. A main tangential velocity term is considered in the governing equations and other tangential velocity terms are ignored. In the governing nonlinear partial differential equations, the derivatives with respect to r, are replaced by fractional derivatives and transformed into a system of nonlinear ordinary differential equations with fractional derivatives using the similarity transformation method and then solved using semi-analytical and numerical methods. The findings show that increasing Re leads to a significant increase in the values of the velocity at the center of the channel, while a slight decrease is observed in the regions near the walls. Increasing Ha leads to a significant decrease in the maximum velocity. Also, the velocity vector lines at fixed radial positions become more uniform and two smaller maximum velocities are created near the channel walls. Increasing the Hartmann number leads to lower concentration throughout the channel, whereas higher Reynolds or Peclet numbers result in reduced concentration at the channel's center. Increasing the order of the fractional derivative of the concentration equation increases the concentration throughout the channel.

Keywords: Converging and diverging channels, injection, concentration, magneto hydrodynamic flow, fractional derivative

1. Introduction

The processes of injection and suction significantly influence flow characteristics, which encompass boundary layer behavior, flow separation, stability management, drag reduction, and the transfer of heat and mass. Suction is recognized as a crucial element in preserving the stability of the boundary layer. In contrast, injection can facilitate the acceleration of instability growth. Both techniques prove effective in minimizing flow separation and reducing drag. Injection can also serve to elevate temperature, while simultaneously functioning as a thermal insulator by establishing an insulating layer along the walls. In fluid mechanics, fractional derivatives are employed as a potent instrument for analyzing magnetohydrodynamic flows and associated chemical reactions. This instrument is crucial for investigating phenomena including free convection, the influence of thermal radiation, the dynamics of porous media modeling, nonlocal and memory effects in turbulence, boundary layer phenomena, and nanoscale heat transfer, which enhances prediction accuracy [1].

Mass diffusion can diverge from the conventional Fickian laws due to nanoscale interactions, a phenomenon known as anomalous diffusion, which manifests as underdiffusion at a reduced rate or superdiffusion at an increased rate compared to classical diffusion. The fractional derivative method is more effective approach for modeling anomalous diffusion [2]. Jeffrey and Hamel were pioneers in analytically solving the flow within convergent-divergent channels [3, 4]. The Jeffrey-Hamel flow is utilized to examine flow instabilities, mass transfer phenomena, and the effects of injection.

This study aims to employ the fractional derivative to explore the influence of a magnetic field and a variable velocity concentrated flow injection from the channel walls on the velocity and concentration distribution within a divergent channel.

2. Governing relations

The divergent channel features an opening angle of

$\alpha = 0/7\text{rad}$ and a radius ranging from $r = 1$ to $r = 4$. The primary flow enters radially without any concentration, while the secondary flow, which is concentrated, is introduced from the walls of the channel. The momentum equation for the incompressible fluid, along with the concentration equation, is articulated as follows [5, 6]:

$$\rho\left(\frac{\partial \vec{V}}{\partial t} + (\vec{V}.\vec{\nabla})\vec{V}\right) = -\vec{\nabla}P + \rho\vec{g} + \vec{\nabla}\cdot\vec{\tau} + \vec{J}\times\vec{B} \tag{1}$$

$$\frac{\partial C}{\partial t} + \vec{V}\cdot\vec{\nabla}C = D\nabla^2 C + S \tag{2}$$

In this context, ρ represents the density, $\vec{V}$ denotes the velocity vector, P signifies the pressure, $\vec{g}$ indicates the gravitational acceleration vector, $\vec{\tau}$ refers to the stress tensor, $\vec{J}$ is the electric current density, $\vec{B}$ represents the magnetic field, C stands for the concentration, D is the mass diffusion coefficient, and S denotes the mass production or decay, where $S = 0$. The primary assumptions are outlined as follows:

1. The fluid under consideration is a Newtonian fluid that is incompressible, with a flow that is two-dimensional and steady; the influence of gravity is considered negligible, and the electric field is assumed to be absent.
2. The radial velocity is prioritized over the angular velocity, leading to the inclusion of a dominant term for the angular velocity in the momentum equations, while other terms are disregarded. It is acknowledged that the tangential velocity varies across the channel, although prior research has typically assumed that the derivative of this velocity along the y or axis is zero.

By utilizing the aforementioned assumptions and removing the pressure, we substitute the first-order derivative concerning r with the Caputo fractional derivative of order β $\left({}_0D_r^{\beta}\right)$ in the momentum equation. Similarly, we replace the first-order derivative with respect to r in the concentration equation with the Caputo fractional derivative of order ξ $\left({}_0D_r^{\xi}\right)$. Furthermore, we employ the following dimensionless similarity transformations:

$$\eta = \frac{\theta}{\alpha},\ \psi = r^{\frac{5}{2}} v_w F(\eta),\ C = r c_w G(\eta) \tag{3}$$

In this context, η represents the dimensionless angular position, while F and G denote the dimensionless functions associated with the momentum equation and the concentration equation, respectively. Additionally, ψ signifies the flow function, and we apply the Caputo fractional derivative relation of the power function [7]:

$${}_0D_r^{\gamma} r^{a} = \frac{\Gamma(a+1)}{\Gamma(a-\gamma+1)} r^{a-\gamma} \tag{4}$$

By utilizing the dimensionless parameters of Reynolds number, Hartmann number, Schmidt number and Peclet number, defined as $\mathrm{Re} = \frac{r^{\frac{5}{2}} v_w}{\nu}$, $\mathrm{Ha} = rB_0\sqrt{\frac{\sigma}{\mu}}$, $\mathrm{Sc} = \frac{\nu}{D}$ and $\mathrm{Pe} = \mathrm{Re}.\mathrm{Sc}$ where v_w is a constant, the following final relations are derived:

$$\begin{aligned} &F^{(IV)} + \frac{5}{2}\alpha \mathrm{Re} F''' F \\ &-\frac{\Gamma(\frac{5}{2})\alpha}{\Gamma(\frac{5}{2}-\beta)}\left(2 - \frac{\frac{5}{2}}{(\frac{5}{2}-\beta)}\right)\mathrm{Re}\, r^{1-\beta} F'' F' + \\ &\left[\begin{matrix} \alpha^2 \Gamma(\frac{5}{2}) r^{-\beta+1}\left(\frac{r^{1-\beta}}{\Gamma(\frac{5}{2}-2\beta)} - \frac{1}{\Gamma(\frac{5}{2}-\beta)}\right) \\ +\alpha^2(1-Ha^2) \end{matrix}\right] F'' \\ &-\frac{4\alpha^3\Gamma(\frac{5}{2})}{\Gamma(\frac{5}{2}-\beta)}\mathrm{Re}\, r^{1-\beta} F' F = 0 \end{aligned} \tag{5}$$

$$\begin{aligned} &G'' + \frac{5}{2}\alpha Pe G' F - \alpha\frac{\Gamma(2)}{\Gamma(2-\xi)} r^{1-\xi} Pe G F' \\ &+\alpha^2\Gamma(2)\left[\frac{1}{\Gamma(2-2\xi)} r^{1-\xi} + \frac{1}{\Gamma(2-\xi)}\right] r^{1-\xi} G = 0 \end{aligned} \tag{6}$$

The boundary conditions are expressed as follows:

$$\begin{cases} \eta = 0 \rightarrow F = -1,\ F' = 1,\ G = 1 \\ \eta = 1 \rightarrow F = 1,\ F' = 1,\ G = 1 \end{cases} \tag{7}$$

To make the concentration dimensionless, the concentration at the initial point of the channel walls is used as a reference value:

$$G = \hat{C} = \frac{C}{C_w|_{r=1}} \tag{8}$$

3. Resolving Systems of Equations

The adaptive fraction technique is employed to address systems of ordinary differential equations. Initially, the linear component of the differential equation is resolved, followed by substituting the variables with polynomial fractional functions [1, 8]. Numerical solutions are obtained using Matlab software.

4. Validation of the solution method

The presence of correlation and consistency between the numerical and analytical solutions substantiates the credibility of the current analytical approach. Conversely, Rahmati and Mollaei [9] conducted a numerical analysis of liquid metal flow within a microchannel featuring a rectangular cross-section subjected to a magnetic field, as illustrated in Figure 1, which depicts the velocity distribution along the walls aligned with the magnetic field direction. An increase in the Hartmann number results in the emergence of an M-shaped velocity distribution within the channel. The rise in the Hartmann number in this study, akin to the findings in [9, 10], leads to the formation of an M-shaped velocity profile, characterized by the presence of two peak velocities near the channel walls. This observation further validates the analytical method employed in this study.

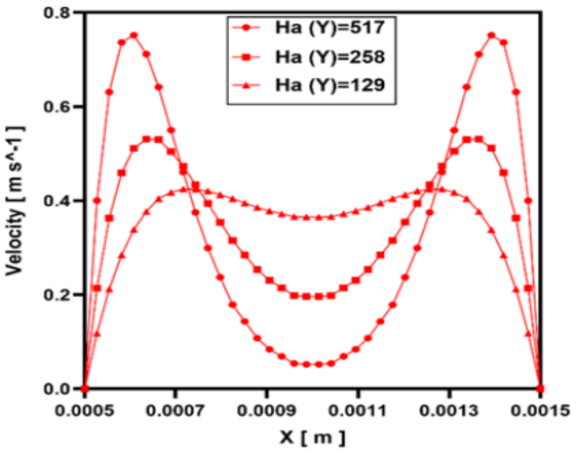

Figure 1. Velocity changes across the channel width in terms of Ha, Rahmati and Mollaei study [9]

5. Results

Figure 2 illustrates that an increase in the Reynolds number results in a notable reduction in the dimensionless concentration at the channel's center, accompanied by a minor rise near the walls. As the Reynolds number increases, there is a concurrent enhancement in both the primary non-concentrated flow and the secondary concentrated flow emanating from the walls. The primary non-concentrated flow contributes to a decrease in concentration, whereas the secondary concentrated flow leads to an increase. Consequently, with a rise in the Reynolds number, one would anticipate a slight elevation in concentration near the walls, where the injection effect is more pronounced. Conversely, at the center of the channel, where the tangential velocity is at its lowest and the primary flow prevails, there is a significant decrease in concentration.

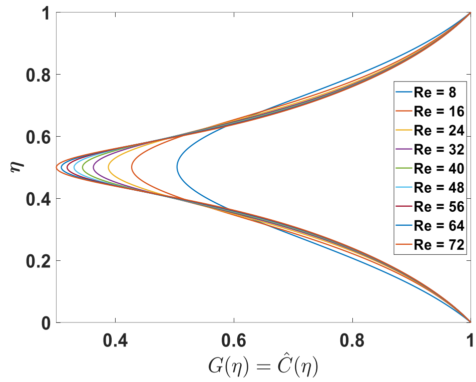

Figure 2. Variations of the dimensionless concentration G in terms of the parameter Re under the condition Ha=2, Sc = 1.5, β = 1, ξ = 1

According to Figure 3, increasing the Hartmann number leads to a slight decrease in the dimensionless concentration across the channel.

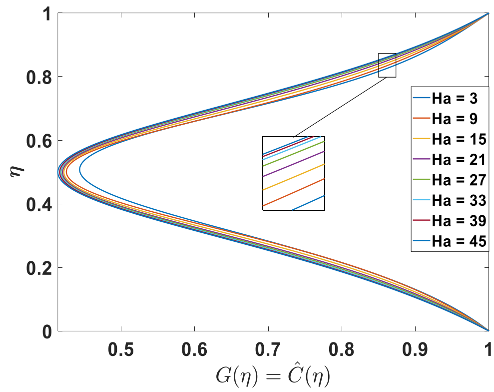

Figure 3. Variations of the dimensionless concentration G in terms of the parameter Ha under the condition Re=16, Sc = 1.5, β = 1, ξ = 1

As illustrated in Figure 4, an increase in results in a rise in the dimensionless concentration throughout the channel.

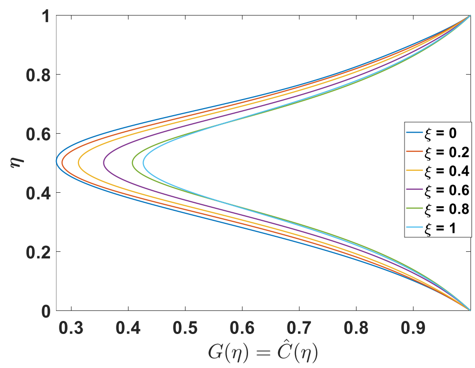

Figure 4. Changes in the G function in terms of the Pe parameter and ξ under the condition Re=16, Ha = 2, β = 1, ξ = 1

6. Conclusions

The primary outcomes of this research are summarized as follows:

• A notable increase in the Reynolds number results in a significant reduction of concentration at the center of the channel; conversely, a minor rise in concentration is noted in areas adjacent to the walls.

• An elevation in the Hartmann number corresponds to a reduction in concentration across the entire channel.

• An increase in the order of the fractional derivative of the concentration equation leads to a rise in concentration throughout the channel.

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