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Presenting a Mathematical Model for Determining the Efficiency of the Fixed Displacement Radial Piston Pump

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Abstract

It is necessary to predict the efficiency of radial piston pumps in different working conditions, in order to optimize energy consumption in power transmission systems of mobile machines. In this article, a mathematical model was presented to estimate the efficiency of the fixed displacement radial piston pump under different working conditions. To evaluate the accuracy of this mathematical model, a flow generation unit, including a radial piston pump and various sensors, was designed and built for the measurement volumetric, mechanical, and total efficiencies, pump flow rate, and input shaft torque. The studies showed that the average difference between the results of experimental measurements of volumetric, mechanical, and total efficiencies and the results of solving the mathematical model are less than 3.5, 3.8, and 4.5%, respectively. The trend of changes in volumetric and total efficiencies of the pump according to the rotational speed of its axis, includes a "dead zone" that expands with the increase of pressure at the pump port. Also, increasing pressure in the outlet of the pump increases the speed corresponding to maximum overall efficiency. However, the mechanical efficiency of the pump shows minimal variation in response to changes in the input shaft's speed and oil pressure.

Keywords: Radial Piston Pump; Volumetric Efficiency; Mechanical Efficiency; Total Efficiency.

1. Introduction

The management of energy consumption in hydraulic power transmission systems in order to reduce the consumption of fossil fuels and reduce environmental effects, has been particularly the focus of researchers in recent years [1]. In the same way, evaluating the efficiency and performance of hydrostatic pumps is very important to reduce energy waste, reduce environmental pollution and fuel consumption in stationary and mobile machines [2]. For this purpose, volumetric and total efficiency diagrams of hydrostatic pumps under specific operating conditions are usually presented by manufacturers in the form of a set of technical charts [3]. In technical manuals, performance curves of hydrostatic pumps are presented for a set of independent application quantities. Therefore, one of the important tools available to the designer, for the proper selection of hydrostatic pumps and motors, based on the considerations and working conditions of hydraulic power transmission systems, are performance charts [4 and 5]. Meanwhile, some manufacturers of hydraulic equipment provide a set of mathematical relationships instead of functional diagrams to determine the volumetric and overall efficiency of hydrostatic pumps [6]. In this case, the volumetric and overall efficiency of hydrostatic pumps are presented in the form of functions in terms of some independent variables, such as the angular velocity of the pump input shaft and the oil pressure in the outlet duct [7]. In recent years, the use of mathematical relationships to estimate the volumetric and overall efficiency of hydrostatic pumps in terms of independent variables, compared to performance diagrams, has been more appreciated and welcomed by the designers of hydraulic power transmission systems. Typically, the optimal selection of hydrostatic pumps for a hydraulic power transmission system is carried out based on performance diagrams and mathematical equations provided by the manufacturing companies, under specific operating conditions [8]. Therefore, in different working conditions, the use of functional diagrams and mathematical relationships presented for choosing hydrostatic pumps will be associated with errors [9].

In the past years, various and diverse researches have been conducted on efficiency analysis and calculation of volumetric and torque losses in hydrostatic pumps. Based on the evaluation and review of the results of various researches in the past years, determining the efficiency of hydrostatic pumps is a complex issue that depends on various and diverse quantities. Typically, mathematical models presented to estimate the volumetric and total efficiency of hydrostatic pumps are obtained based on rewriting the energy conservation relations governing the control volumes corresponding to the suction and delivery regions [8]. In this paper, considering the weaknesses of the existing mathematical and numerical models, a different mathematical model based on the performance diagrams of hydrostatic pumps is presented. This mathematical model, unlike other existing models, does not depend on the internal structure of the hydrostatic pump. Also, it is easily possible to use this mathematical model to estimate the mechanical, total and volumetric efficiencies, the pump outlet oil flow rate and the torque required for the rotational movement of its input shaft, under different working conditions. In this article, after presenting a mathematical model to estimate the functional characteristics of PFR525 radial piston pump (volumetric, total and mechanical efficiencies, output oil flow rate from the pump and torque required for the rotational movement of its input axis), design and construction of the flow generation unit is done to evaluate the correctness of the mathematical model solution and compare them with experimental results.

2. Methodology

The volumetric efficiency of a hydrostatic pump (η_p^v), according to ISO 4391: 1983, is defined as follow [1]:

$$\eta_p^v = \frac{D_p\,\omega_p - q_{pL}}{D_p\,\omega_p} \tag{1}$$

Studies show that the total volume loss in a hydrostatic pump due to oil leakage (q_{pL}) is a function of the oil pressure difference in the pump passage [8]:

$$q_{pL} = K_s \Delta \mathrm{p} \tag{2}$$

Therefore, the volumetric efficiency of the pump is rewritten as follows:

$$\eta_p^v = 1 - \phi\left(\frac{\Delta p^*}{D^* \omega^*}\right) \tag{3}$$

The quantities D^*, ω^* and Δp^* are the relative displacement volume, relative angular velocity and relative pressure difference, respectively. In order to provide a mathematical model to determine the volumetric efficiency of the fixed displacement radial piston pump, model PFR522, it is necessary to measure the volumetric efficiency at four specific nodes, at two different pressure differences of 100bar ($\Delta p^* = 0.4$) and 250bar ($\Delta p^* = 1$), and also at two specific rotational speeds. Accordingly, for a pressure ratio of 0.4, in two speed ratios of 0.137 (Node No. 1) and 1 (Node No. 2), the volumetric efficiency of the PFR522 pump was measured as 0.884 and 0.903, respectively. Similarly, the volumetric efficiency of the PFR522 pump, at unit pressure ratio, at two speed ratios of 0.137 (node number 3) and 1 (node number 4), is estimated to be 0.884 and 0.903, respectively. Then, based on the measured data, the quantity ϕ is obtained in terms of relative velocity (ω^*) and relative pressure difference (Δp^*) for the PFR522 radial piston pump:

$$\phi_{\mathrm{pump}} = 0.0071 + 0.2976\omega^* + 0.0009\Delta p^* - 0.1567\omega^*\Delta p^* \tag{4}$$

Finally, by placing the numerical values obtained from relation 5 in relation 4, the volumetric efficiency of the pump is obtained corresponding to the assumed working conditions.

In other words, the set of relations 3 and 5 are used to estimate the volumetric efficiency of PFR522 radial piston pump.

On the other hand, for mathematical modeling of the overall efficiency of hydrostatic pumps, it is necessary to rewrite the relationships related to mechanical efficiency in terms of the rotational axis speed and oil pressure difference. The mechanical efficiency of hydrostatic pumps (η_p^m), according to ISO 4391:1983, is defined as follows [1]:

$$\eta_p^m = \frac{1}{1 + \phi_\omega\left(\frac{\omega^*}{\Delta p^* D^*}\right) + \frac{\phi_f}{\Delta p^* D^*}} \tag{5}$$

Thus, the total pump efficiency is obtained from the product of the volumetric efficiency (η_p^v) and the mechanical efficiency (η_p^m):

$$\eta_p = \frac{\eta_p^v}{1 + \phi_\omega\left(\frac{\omega^*}{\Delta p^* D^*}\right) + \frac{\phi_f}{\Delta p^* D^*}} \tag{6}$$

In order to provide a mathematical model to determine the total efficiency of the fixed displacement piston pump, model PFR522, in two different pressure differences 100bar ($\Delta p^* = 0.4$) and 250bar ($\Delta p^* = 1$), the total efficiency is measured at four specific speeds. Therefore, for a pressure ratio of 0.4, the overall efficiency of the PFR522 pump was measured to be 0.832, 0.839, 0.824, and 0.811 at speed ratios of 0.137 (node number 1), 0.41 (node number 2), 0.689 (node number 3), and 1 (node number 4), respectively. Similarly, for a unit pressure ratio, at speed ratios of 0.137 (node number 1), 0.41 (node number 2), 0.689 (node number 3), and 1 (node number 4), the overall efficiency of the PFR522 pump was calculated to be 0.767, 0.808, 0.807, and 0.803, respectively. Then, based on the measured data, the two quantities ϕ_ω and ϕ_f, in Equation 7, are obtained in terms of relative velocity and relative pressure difference for the PFR522 radial piston pump, as follows:

$$\begin{aligned}\phi_\omega = {} & -0.0485 - 2.3\times10^{-3}\omega^* - 0.02\Delta p^* \\ & +0.283\Delta p^*\omega^* + 9.26\times10^{-3}(\omega^*)^2 \\ & +0.023(\omega^*)^2\Delta p^*\end{aligned} \tag{7}$$

$$\phi_f = 0.0461 + 0.041\Delta p^* \tag{8}$$

Finally, the total efficiency of the hydrostatic pump corresponding to each relative speed of the input shaft (ω_i^*) and at any given pressure difference (Δp_j^*) is obtained:

$$\eta_{pi} = \frac{\eta_{pi}^{v}}{1 + \phi_{\omega i}^{j}\left(\frac{\omega_i^*}{\Delta p_j^* D^*}\right) + \frac{\phi_{fi}^{j}}{\Delta p_j^* D^*}} \tag{9}$$

In order to experimentally evaluate and check the correctness of the results of solving the mathematical model to determine the total efficiency, volumetric efficiency and mechanical efficiency of PFR522 radial piston pump, the design and construction of the flow generation unit, in Figure 1, is done.

3. Discussion and Results

In Figure 2, the trend of changes in the volumetric efficiency of the PFR522 radial piston pump corresponding to pressure ratios of 0.4 and 1, in terms of the input shaft rotational speed ratio, obtained from the set of equations 3 and 4, is plotted. In order to evaluate the accuracy of the mathematical model presented for estimating the volumetric efficiency of the PFR522 radial piston pump, in addition to the theoretical results, the results of experimental measurements are also shown in Figure 2.

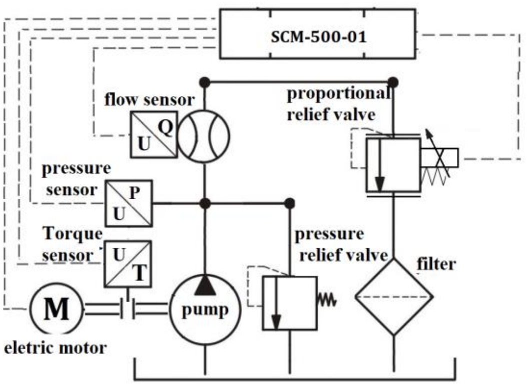

Figure 1. Oil flow generation unit including PFR522 radial piston pump

In this way, it is possible to easily check the accuracy of the results obtained from solving the mathematical equations. According to Figure 2, the results obtained from the mathematical equations are in good agreement with the experimental results measured by the system shown in Figure 1. Investigations show that the results of solving the mathematical model differ by 3.3% and 3.7% on average with the experimental results for pressure ratios of 0.4 and 1.

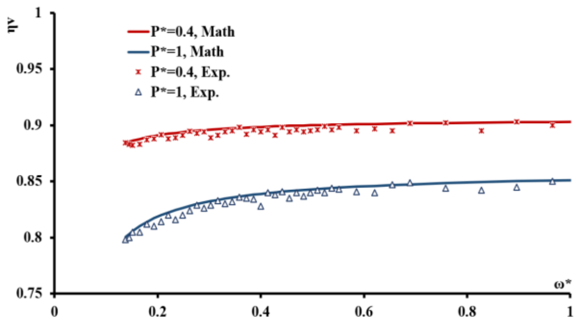

Figure 2. Trend of changes in volumetric efficiency of PFR522 radial piston pump according to input shaft rotational speed at pressure ratios of 0.4 and 1

Also, the process of changes in the total efficiency of the PFR522 radial piston pump, in terms of the rotational speed of the drive motor output shaft, in the pressure ratios of 0.4 and 1, is determined and shown in Figure 3. In order to evaluate the accuracy of the mathematical model, in Figure 3, the results of measuring the total efficiency of PFR522 pump and the results of solving the mathematical model are given. Investigations show that the results of solving the mathematical model differ from the experimental results at pressure ratios of 0.4 and 1 by an average of 4.3 and 4.7 percent. The trend of changes in the total efficiency of the PFR522 radial piston pump corresponding to pressure ratios of 0.2, 0.4, 0.6, 0.8 and 1, in a wider range of rotational speed of the output shaft of the driving motor, is shown in Figure 4.

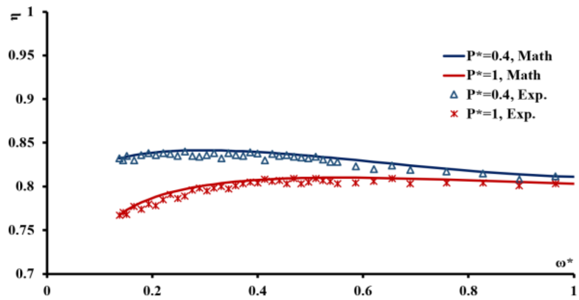

Figure 3. Trend of changes in total efficiency of PFR522 radial piston pump according to input shaft rotational speed at pressure ratios of 0.4 and 1

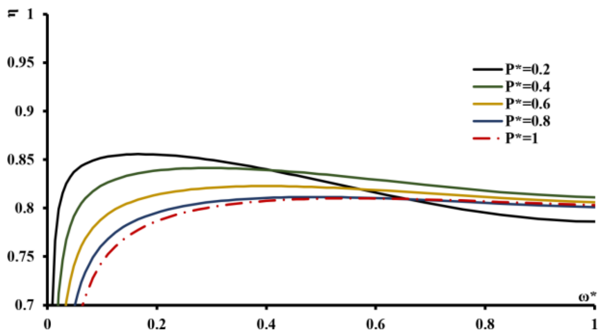

Figure 4. Trend of changes in total efficiency of PFR522 radial piston pump according to input shaft rotational speed at different pressure ratios

4. Conclusions

In this article, a mathematical model was presented to predict the performance of hydrostatic pumps in hydraulic power transmission systems and determine their volumetric, mechanical, and overall efficiencies. Accordingly, optimal selection of hydrostatic pumps in hydraulic power transmission systems, as well as energy consumption management in them, will be easily possible. A summary of the results of this research is as follows:

- The average difference between the results of experimental measurements and the results of solving the mathematical model of dependent quantities, including volumetric efficiency and total efficiency, was determined to be less than 3.5 and 4.5 percent, respectively.
- The trend of changes in volumetric and total efficiencies of a radial piston pump in terms of the rotational speed of the input shaft, at different pressure ratios, due to the presence of oil leakage, includes a dead zone.
- The diagram of the total efficiency of the radial piston pump will be maximum in a certain range of input shaft speed.

5.References

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