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Synchronization of Uncertain Multiple Chaotic Systems Based on an Optimal Nonlinear Observer: A Secure Communication Approach

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Abstract

This paper investigates modified projective and transmission synchronization in the presence of uncertainty and disturbance signals using an optimal observer-based super-twisting approach for multiple chaotic systems involving one drive system and two response systems. To estimate the uncertain terms and quickly deal with the disturbances, an optimal finite-time super-twisting observer is designed. Then, a finite-time controller is designed to ensure chaos synchronization by driving observer error systems to zero based on the Lyapunov stability theorem. The proposed control law can be used in chaotic secure communication system to increase security. Furthermore, we suggest the cascade inclusion method, which is more secure than the previous approaches, for use in secure communication applications. Finally, the simulation results demonstrate the robustness and effectiveness of the suggested approach when subjected to uncertainty and disturbances.

Keywords: Multiple Chaotic Systems, Super-twisting Observer, Uncertain Dynamics, Synchronization, Secure Communication, Stability Analysis, Optimization.

1. Introduction

The synchronization of chaotic systems has become a cornerstone in modern control theory, particularly for applications requiring secure data transmission and encryption. These systems, characterized by extreme sensitivity to initial conditions and complex nonlinear dynamics, offer unique advantages for information security but also present significant control challenges [1]. In recent years there has been a growing interest in multi-chaotic system synchronization, driven by demands for enhanced communication security and robust control in noisy environments [2].

Traditional synchronization approaches, including adaptive control [3] and sliding mode techniques [4], have demonstrated limitations in effectively handling simultaneous parametric uncertainties and external disturbances. While adaptive methods can estimate unknown parameters, they often exhibit slow convergence rates. Sliding mode controllers, though robust, suffer from inherent chattering phenomena that can degrade system performance [5]. Observer-based methods have emerged as promising alternatives; however, existing designs frequently fail to guarantee both finite-time convergence and optimal disturbance rejection [6]. The current state-of-the-art reveals three critical gaps: (1) Most synchronization schemes consider only matched uncertainties, neglecting realistic scenarios involving unmatched disturbances; (2) finite-time stability is rarely achieved without compromising control smoothness; and (3) existing secure communication protocols remain vulnerable to advanced eavesdropping techniques [7]. These limitations underscore the need for our integrated approach combining advanced estimation, optimal control, and cryptographic innovation. This paper makes four significant contributions to address these challenges:

- A super-twisting observer with finite-time convergence is developed to simultaneously estimate system uncertainties and external disturbances.
- A Lyapunov-based control law is derived to guarantee synchronization within a predefined time, addressing the slow convergence issues in existing methods [5].
- The genetic algorithm (GA) optimizes controller parameters, minimizing synchronization error and enhancing transient performance.
- The cascade inclusion method is introduced, combining multiple chaotic signals to significantly improve communication security against eavesdropping.

These contributions are validated through rigorous simulations and comparative analysis, demonstrating superior performance over state-of-the-art techniques.

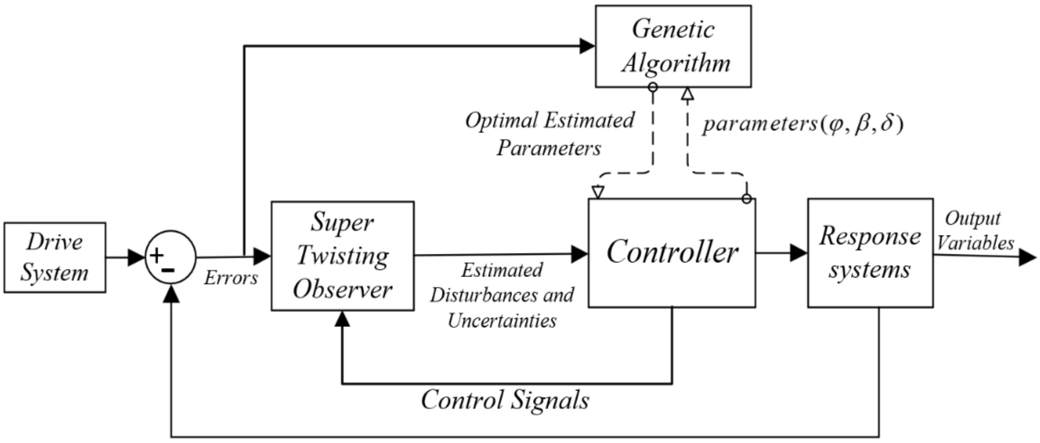

Fig 1. The block diagram of the proposed control framework

2. Methodology

The drive and response systems are modeled as:

$$\dot{x}_{1i} = q_{1i}(x_1) + \Delta Q_{1i}(x_1) + b_{1i}(t) \quad (1)$$

$$\dot{x}_{ni} = q_{ni}(x_n) + u_{ni} + \Delta Q_{ni}(x_n) + b_{ni}(t) \quad (2)$$

for $n = 2{,}3$. Note that ΔQ represents uncertainties and $b(t)$ disturbances. The key components of the proposed control framework are provided in what follows.

- Super-twisting observer:

$$\dot{\hat{e}}_{ni} = q_{1i}(x_1) - \omega_{ni} q_{ni}(x_n) + \hat{B}_{ni} - \omega_{ni} u_{ni}$$
$$-\psi_{ni}|\hat{e}_{ni} - e_{ni}|^{0.5} sign(\hat{e}_{ni} - e_{ni}) \quad (3)$$
$$\dot{\hat{B}}_{ni} = -\kappa_{ni} sign(\hat{e}_{ni} - e_{ni})$$

- Finite-time controller:

$$u_{ni} = \frac{1}{\omega_{ni}}(q_{1i} + \hat{B}_{ni}) - q_{ni}$$
$$+\frac{1}{\omega_{ni}}\left(\phi_{ni} e_{ni} + \beta_{ni}|e_{ni}|^{\delta} sign(e_{ni})\right) \quad (4)$$

- Stability analysis: Using the Lyapunov function $V_1 = \zeta^T P \zeta$, where $\zeta^T = [|\tilde{e}_{ni}|^{0.5} sign(\tilde{e}_{ni}), \tilde{B}_{ni}]$, we derive convergence condition as

$\psi_{ni} > 1,$

$$\kappa_{ni} > \frac{-\psi_{ni}(E_i^2 + 3) + E_i^2 + 3\psi_{ni}^2}{\psi_{ni}^2 + \psi_{ni}} \quad (5)$$

- Secure communication protocol: The cascade inclusion method enhances security through encryption process and decryption process. Each process is described as:

(a) Encryption process

Message embedding:

$$m(t) \to Y_1 = x_{11} + \alpha m(t) \quad (6)$$

Chaotic masking: $Y = Y_1 + Y_2$, where Y_2 is obtained from secondary chaotic system.

(b) Decryption process

Synchronization: $\lim_{t\to\infty} \|x_r - x_t\| = 0$

Message recovery: $\hat{m}(t) = \frac{Y - Y_2}{\alpha}$

The block diagram of the proposed controller is also depicted in Fig. 1 while the cascade inclusion approach [8] is shown in Fig. 2.

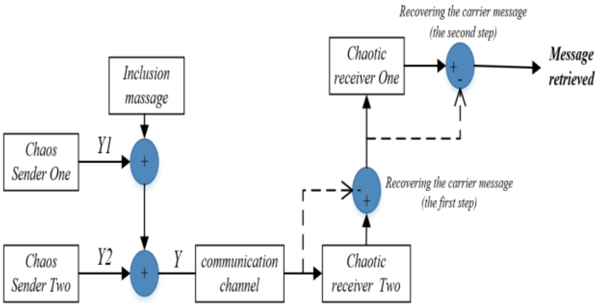

Fig 2. The diagram of cascade inclusion approach

3. Discussion and Results

The paper employs a super-twisting observer (STO) combined with GA to achieve robust synchronization of uncertain chaotic systems. The STO provides finite-time convergence for estimating disturbances and uncertainties, eliminating chattering effects typical of traditional sliding-mode observers. By integrating GA, the controller parameters are optimized to minimize synchronization error while ensuring stability. This hybrid approach enhances both speed and precision, as GA fine-tunes the STO's performance, reducing transient oscillations and accelerating convergence.

The cascade inclusion approach proposed in this paper enhances secure communication by combining two chaotic systems—a multiple chaotic system and the Chen system—into a single, more complex dynamic structure. By embedding the message signal within one chaotic system and then cascading it with another, the resulting output exhibits significantly higher complexity in phase space, making it extremely difficult for attackers to extract or predict the hidden information. This method not only preserves the chaotic nature of the carrier signals but also introduces an additional layer of security compared to traditional chaotic masking or modulation techniques. Numerical simulations confirm that the cascade inclusion method effectively obscures the message, even under noise interference, demonstrating its robustness for practical secure communication applications.

Numerical results confirm that the optimized STO outperforms conventional methods, achieving near-zero error in synchronization even under external disturbances. Due to figure limitations, only key synchronization and estimation plots are shown in Fig. 3 and Fig. 4. While additional plots (e.g., phase portraits) would further support the claims, the included figures suffice to validate the core contributions.

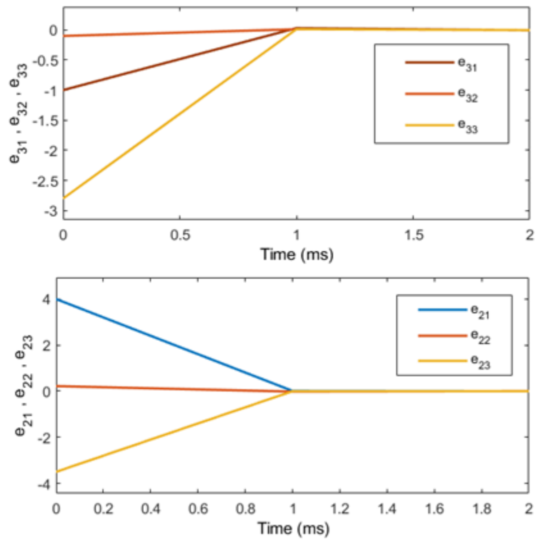

Fig. 3. Synchronization errors of second and third response systems

4. Conclusions

The paper concludes by presenting a robust framework for synchronizing multiple uncertain chaotic systems under disturbances, with applications in secure communication. Key contributions include an optimal super-twisting observer for finite-time estimation of uncertainties, a Lyapunov-based controller ensuring precise synchronization with reduced chattering, and a novel cascade inclusion method that enhances security by merging chaotic signals. Genetic algorithm optimization further refines controller parameters, minimizing synchronization error. Simulations validate the method's effectiveness in handling disturbances and improving communication security. Future work will extend this approach to fractional-order chaotic systems, building on the demonstrated advancements in robustness and encryption complexity.

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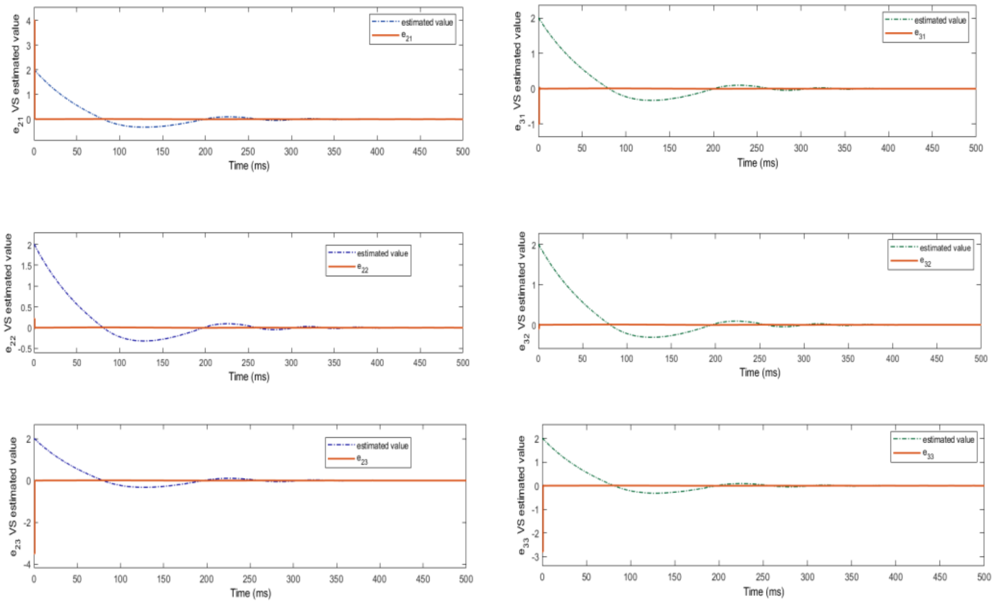

Fig. 4. The estimation of response system errors