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Fluid-Structure Interaction problem in the framework of Arbitrary Lagrangian-Eulerian (ALE) description using a Monolithic approach in 2DAbstract

M.A. Jahangiri[1], R. Attarnejad[2]

[1] M.S.c. Student, College of Engineering, School of Civil Engineering, University of Tehran, Tehran, Iran
[2] Professor, College of Engineering, School of Civil Engineering, University of Tehran, Tehran, Iran
* Corresponding author: attarnjd@ut.ac.ir

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Abstract

The partitioned approach for solving fluid-structure interaction problems is prone to numerical instabilities, often leading to a lack of convergence. Overcoming these challenges requires stabilization techniques and reduced time steps, significantly increasing computational costs. In this study, a monolithic formulation within the Arbitrary Lagrangian-Eulerian (ALE) framework is proposed for analyzing fluid-structure interaction problems, enabling efficient tracking of moving boundaries. The Navier-Stokes equations for unsteady fluid flow and the linear elasticity equations for the structure are solved in a strongly coupled manner. Comparison with the partitioned approach revealed that the average computational time per step in the partitioned method was 51 seconds, while the proposed approach required only 7 seconds, demonstrating its computational efficiency. Furthermore, the proposed method eliminates the added mass effect, enhances solution accuracy, and prevents sudden oscillations observed in the partitioned approach.

Additionally, mesh dependency analysis showed that increasing the degrees of freedom from 85,452 to 1,141,027 resulted in only a 2% increase in pressure and displacement, indicating minimal sensitivity to mesh size. This highlights the robustness and efficiency of the proposed method in solving fluid-structure interaction problems.

Keywords: Fluid-Structure Interaction, Navier-Stokes Equation, Linear Elasticity, Arbitrary Lagrangian-Eulerian (ALE), Monolithic approach

1. Introduction

Fluid-structure interaction (FSI) problems are relevant in various engineering fields, including aerospace, biomedical engineering, and mechanical engineering, with applications such as airflow around aircraft, blood flow in arteries, and wind turbine blade interactions. Traditional FSI solvers are categorized into two main approaches: the partitioned method, where fluid and structural domains are solved separately with data exchange, and the monolithic method, which simultaneously solves fluid and structure equations in a fully coupled manner. While the partitioned approach is computationally expensive and sensitive to added mass effects, the monolithic approach improves stability, speed, and convergence. A major computational challenge in FSI problems is tracking mesh motion and deformation, which is effectively addressed by the Arbitrary Lagrangian-Eulerian (ALE) formulation, as it combines Eulerian and Lagrangian descriptions to enable a moving mesh while minimizing numerical diffusion. Multiphysics problems, such as fluid-structure interaction (FSI), are significant topics with widespread applications in engineering sciences. Some notable examples include aircraft wings exposed to wind loads in aerospace engineering [1,2], blood flow modeling in biomechanical engineering [3], wind flow in wind turbines [4], lubricating fluid between bearings and gears in the automotive industry [5], and wave-structure interactions in coastal, port, and offshore engineering [6,7]. The latter encompasses structures such as bridge piers, dams, breakwaters, coastal protection structures, ship hulls, and more.

The inherent challenges in fluid-structure interaction problems have led to the development of various numerical methods, which can be categorized into partitioned and monolithic approaches based on how the governing equations of the fluid and structure interact [8]. If the coupling of the fluid and structural equations occurs implicitly within a single system of equations at each time step, the solution strategy follows a monolithic approach, where the fluid and structural domains are considered as a continuous domain [9].

The monolithic approach offers greater solution stability compared to the partitioned approach and also provides higher accuracy and computational efficiency [10]. However, due to the lower complexity of

discretizing the fluid and structural domains separately, most engineering simulation software [11] employs the partitioned approach [12].

In the partitioned approach, the FSI problem is decomposed into separate subproblems for the fluid and structural domains. These domains are modeled with distinct computational grids and are analyzed separately using numerical methods. Subsequently, the boundary conditions arising from fluid-structure interaction are explicitly transferred between the solvers [13]. One of the drawbacks of this method is the handling of interface forces, which can lead to numerical instability and solution divergence. Thus, to ensure solution stability and convergence, a relatively large number of iterations is required [14].

In the weak coupling method, only the effect of the fluid on the structure is considered. Although this method is computationally less expensive, it is less accurate than the strong coupling method, making it less favorable. On the other hand, the strong coupling method, due to its bidirectional interaction, can introduce numerical instabilities caused by the added mass effect, which must be accounted for in the analysis [15].

The phenomenon of added mass effect in the strong coupling approach occurs when the density of the structure is equal to or greater than the density of the fluid. A real-life example of the same exists in biomechanical engineering, where the structural part is embodied by blood vessels and the fluid part is embodied by blood [16]. Experiments conducted by Degroote et al. [13] and Küttler et al. [17] have revealed that monolithic schemes are more effective in comparison to partitioned schemes. Furthermore, they demonstrated that the single implementation would remove the numerical instabilities caused by the added mass effect present in partitioned methods, as both the fluid and structure governing equations are solved simultaneously in one system at every time step. Neglect of the added mass effect in partitioned techniques can result in inaccurate predictions of the system dynamics, particularly in cases involving large deformations [14,18,19].

The Eulerian and Lagrangian perspectives [20] are the fundamental concepts of the numerical simulation of movement. In the Lagrangian (material) description, the mesh nodes follow the material particles. The drawback of this technique is the potential for instability in the simulation of large deformations [21]. In the Eulerian (spatial) description, the mesh nodes remain fixed. One of the major advantages of this approach is that it can deal with large deformations without mesh displacement; however, it creates convective terms, which are one of the sources of numerical instability [22].

Some techniques have been proposed by many researchers in the engineering literature to enhance numerical solution accuracy for partial differential equations (PDEs) in dynamic problems. One of the most popular techniques is the Arbitrary Lagrangian-Eulerian (ALE) formulation, which was first presented in the early 1980s by Hughes et al. [23] and Donea et al. [24]. The method is based on the establishment of a suitable mapping (arbitrary) of the prescribed reference configuration onto the current dynamic field. The Arbitrary Lagrangian-Eulerian (ALE) formulation is an intermediate and generalized approach, taking into account the advantage of both the Eulerian and Lagrangian descriptions simultaneously.

2. Methodology

For a Newtonian incompressible fluid, the Navier-Stokes equations in the Eulerian formulation, neglecting body forces, are expressed on the grounds of conservation of mass and momentum as follows:

$$\rho_f \left(\frac{\partial \mathbf{v}_f}{\partial t} + (\mathbf{v}_f \cdot \nabla)\mathbf{v}_f \right) = -\nabla p + \mu_f \nabla^2 \mathbf{v}_f \quad \text{in } \Omega_t^f \tag{1}$$

$$\nabla \cdot \mathbf{v} = 0 \quad \text{in } \Omega_t^f \tag{2}$$

In which, ρ_f, μ_f, $\mathbf{v}_f$, and p are representatives of fluid density, dynamic viscosity, velocity vector, and fluid pressure, respectively. The right-hand side of the equation is obtained under the assumption of a Newtonian fluid, while Equation (2) is considered as the incompressibility condition [25]. Kinematic viscosity is also represented by the formula $v_f = \mu_f / \rho_f$.

The equation of elasticity in the deforming domain, without body forces, is the following:

$$\rho_s \frac{\partial^2 u}{\partial t^2} = \nabla \cdot \sigma_s(u) \quad in \ \Omega^s \tag{3}$$

where ρ_s and u are the density and displacement of the solid, respectively. The stress tensor σ_s is given by:

$$\sigma_s = 2\mu_s \epsilon(u) + \lambda \nabla . \big(\epsilon(u)\big) I \tag{4}$$

These constants λ and μ_s are referred to as the Lamé parameters. Moreover, the strain tensor of the solid under the assumption of small deformations can be written as:

$$\epsilon = \frac{1}{2}(\nabla u + (\nabla u)^T) \tag{5}$$

Given:

$$u = u^0 + \int_0^t \mathbf{v} d\tau \tag{6}$$

The equation for the stress tensor stress is remodeled as:

$$\sigma_s(\boldsymbol{v}_s) = 2\mu_s\epsilon\left(\int_0^t \boldsymbol{v}_s d\tau\right) + \lambda\nabla.\left(\epsilon\left(\int_0^t \boldsymbol{v}_s d\tau\right)\right)I \quad (7)$$

ALE offers a hybrid Eulerian-Lagrangian method, in which the Eulerian method, which has a fixed mesh, is optimally applied to fluid dynamics, and the Lagrangian method, which has a mesh that moves with the material, is optimally applied to structural analysis.

The mapping function relates the reference and deformed configurations through the relation β(X, t) = (x, t), where the Jacobian is J = ∂x/∂X. The convective velocity form, C = v − w, allows accurate tracking of moving boundaries, where w is the mesh velocity. To avoid excessive distortion, a mesh update equation, $\nabla^2 u = 0$, is included, which allows smooth mesh motion.

Numerical discretization of the FSI problem is accomplished through the application of the Finite Element Method (FEM) for spatial discretization, employing Taylor-Hood elements (P2-P1) for velocity and pressure. The Backward-Euler scheme is employed to deal with time integration. The weak form of the governing equations is applied to ensure accuracy and stability of the numerical solution.

Weak form of governing equations for fluid domain:

$$\begin{aligned}&\frac{\rho_f}{\Delta t}\left(\mathbf{v}_f{}^{n+1},\varphi\right)_{\Omega_t^f}\\&+\rho_f\left(\left(\left(\mathbf{v}_f{}^{n+1}-\mathbf{w}_f\right).\nabla\right)\mathbf{v}_f{}^{0},\varphi\right)_{\Omega_t^f}\\&-(p,\nabla.\varphi)_{\Omega_t^f}+2\mu_f\left(\varepsilon\left(\mathbf{v}_f{}^{n+1}\right),\nabla\varphi\right)_{\Omega_t^f}\\&=\frac{\rho_f}{\Delta t}\left(\mathbf{v}_f{}^{n},\varphi\right)_{\Omega_t^f}\end{aligned} \quad (8)$$

$$-\left(\nabla.\mathbf{v}_f{}^{n+1},\eta\right)_{\Omega_t^f}=0 \quad (9)$$

$$\Delta t\left(\nabla\mathbf{w}_f,\nabla\psi\right)_{\Omega_f}=-(\nabla u^n,\nabla\psi)_{\Omega_f} \quad (10)$$

Concerning the solid domain:

$$\begin{aligned}&\frac{\rho_s}{\Delta t}(\mathbf{v}_s{}^{n+1},\varphi)_{\Omega_s}\\&+\rho_s\left((\mathbf{v}_s{}^{n+1}.\nabla)\mathbf{v}_s{}^{0},\varphi\right)_{\Omega_s}\\&+\Delta t2\mu_s\left(\varepsilon(\mathbf{v}_s{}^{n+1}),\varepsilon(\varphi)\right)_{\Omega_s}\\&+\Delta t\lambda_s(\nabla.\mathbf{v}_s{}^{n+1},\nabla.\varphi)_{\Omega_s}\\&=\frac{\rho_s}{\Delta t}(\mathbf{v}_s{}^{n},\varphi)_{\Omega_s}-2\mu_s\left(\varepsilon(u^n),\varepsilon(\varphi)\right)_{\Omega_s}\\&-\lambda_s(\nabla.u^n,\nabla.\varphi)_{\Omega_s}\end{aligned} \quad (11)$$

$$\frac{1}{\delta}(\mathbf{v}_s{}^{n+1},\psi)_{\Omega_s}-\frac{1}{\delta}(\mathbf{w}_s,\psi)_{\Omega_s}=0 \quad (12)$$

These are solved simultaneously in the monolithic framework.

3. Discussion and Results

Case one is employed as a reference example for fluid-structure interaction (FSI) issues, in which an elastic body is immersed in a two-dimensional channel with material properties, geometry, and boundary conditions that are the same as those presented in reference [26]. In this case, validation has been conducted so as to enable comparison study between the coupled method and the discretized method. Figure 1 illustrates the relationship between iterations and computational time for case 1 at an inlet velocity of 1 m/s. The x-axis is utilized to represent the frequency at which the equations have been solved in order to achieve a converged solution, and the y-axis represents each iteration's computational time in seconds. The computational time taken for each step in the partitioned approach was on average 51 seconds, while that for each step in the monolithic approach was 7 seconds. The graph shows that computer time per iteration for the partitioned approach is much larger than that for the monolithic approach. This is because, in each step of the partitioned approach, equations are solved multiple times until convergence is achieved, whereas in the monolithic approach, a single system of equations is solved at a time per iteration.

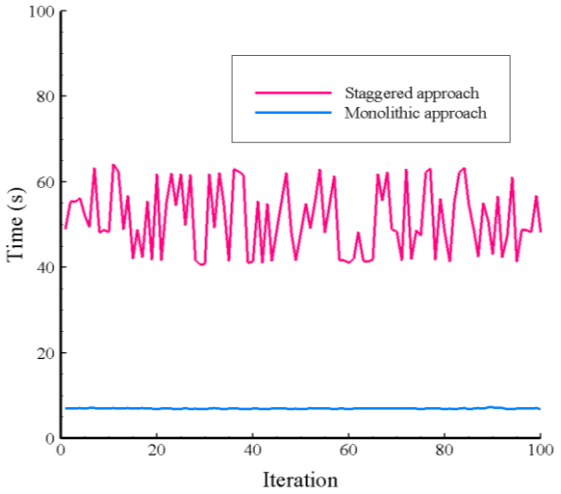

Figure 1. Number of Iterations versus Computational Time.

In the second case, to analyze the solution independence from the computational mesh, the displacement at the free end of the solid structure and the pressure on the left boundary of the structure were analyzed for three different mesh configurations, as presented in Table 1.

Table 1. Types of mesh

	Degree of freedom	Number of fluid elements	Number of solid elements
Mesh 1	1141027	106671	26984
Mesh 2	293312	27095	7120
Mesh 3	85452	7174	2760

The results presented in Figures 2 and 3 show that these values are consistently similar in spite of changing mesh size, which implies that the results are converging and independent of mesh resolution.

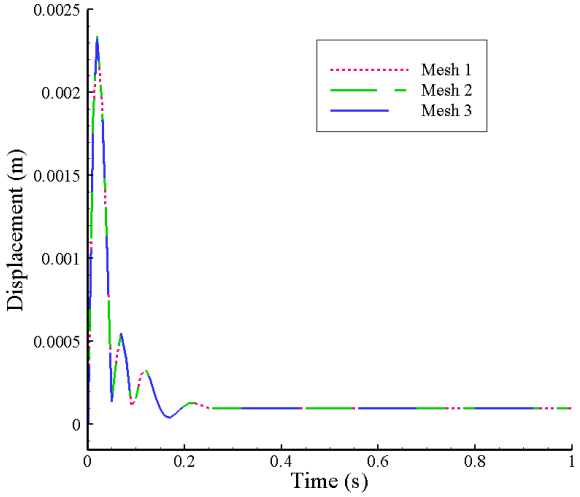

Figure 2. Displacement

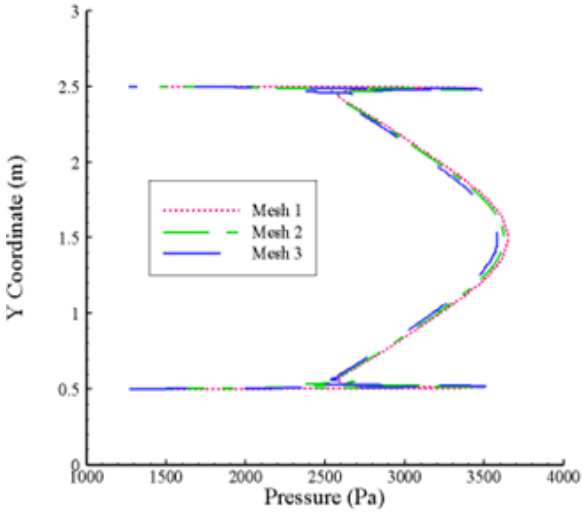

Figure 3. Pressure on the left boundary of the structure

Additionally, as shown in Figure 4, the highest velocity of 1.5036 m/s is observed at the top and bottom zones of the structure.

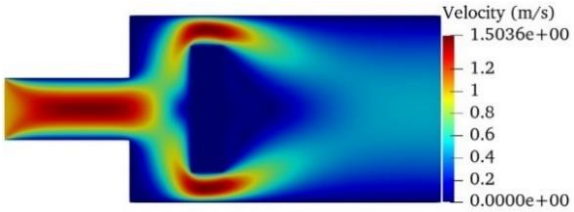

Figure 4. Velocity contour

4. Conclusions

The research explored fluid-structure interaction (FSI) issues through an integrated approach in the Arbitrary Lagrangian-Eulerian (ALE) formalism, which was realized in the FEniCS computing environment. A stable coupling technique for the simultaneous solution of both fluid and structural equations allowed for better modeling of the interaction between the two domains, enhancing both convergence and numerical stability.

The findings presented in the first case indicated that this method, while decreasing the computation time for each iteration, was more accurate than partitioned methods. On average, each iteration of the partitioned method required 51 seconds, whereas each iteration using the integrated method required only 7 seconds.

Furthermore, both numerical and parametric studies, including the influence of the Reynolds number and mesh sensitivity analysis, validated the efficiency of the proposed method for different conditions. For a Reynolds number of 70, the coupled method was shown to be superior to the partitioned algorithm and successfully yielded convergent and stable solutions by cancelling out the added mass effect.

Numerical solutions have demonstrated that the current code, like other comparable studies, yields accurate results for Reynolds numbers as high as 100.

Furthermore, a mesh dependency study revealed that doubling the degrees of freedom from 85,452 to 1,141,027 resulted in insignificant changes in pressure (from 3605 to 3650 Pascals) and displacement (from 0.0232 to 0.0235 meters), and hence an insignificant dependency on mesh resolution.

The comparison of the computational costs between this and the partitioned method confirmed its superiority in both lowering computational time and improving solution quality. This strategy has the capability of being an effective method for handling challenging fluid-structure interaction issues in engineering applications, while it can also be extended for dealing with large deformation problems in a way to avoid numerical instability.

5. References

[1] C. Farhat, K. G. Van der Zee, and P. Geuzaine, "Provably second-order time-accurate loosely-coupled solution algorithms for transient nonlinear computational aeroelasticity," Computer methods in applied mechanics and engineering, vol. 195, no. 17-18 ,pp. 1973-2001, 2006.

[2] J. Hoffman, J. Jansson, and N. Jansson, "FEniCS-HPC: Automated predictive high-performance finite element computing with applications in aerodynamics," in Parallel Processing and Applied Mathematics: 11th International Conference ,PPAM 2015, Krakow, Poland, September 6-9, 2015. Revised Selected Papers, Part I 11, 2016: Springer, pp. 356-365 .

[3] G. Link, M. Kaltenbacher, M. Breuer, and M. Döllinger, "A 2D finite-element scheme for fluid–solid–acoustic interactions and its application to human phonation," Computer Methods in Applied Mechanics and Engineering, vol. 198, no. 41-44, pp. 3321-3334, 2009.

[4] Y. Bazilevs, M. C. Hsu, J. Kiendl, R. Wüchner, and K. U. Bletzinger, "3D simulation of wind turbine rotors at full scale. Part II: Fluid–structure interaction modeling with composite blades," International Journal for numerical methods in fluids, vol. 65, no. 1-3, pp. 236-253, 2011.

[5] R. Castilla, P. Gamez-Montero, N. Ertürk, A. Vernet, M. Coussirat, and E. Codina, "Numerical simulation of turbulent flow in the suction chamber of a gearpump using deforming mesh and mesh replacement," International Journal of Mechanical Sciences, vol. 52, no. 10, pp. 1334-1342, 2010.

[6] R. K. Jaiman, F. Shakib, O. H. Oakley Jr, and Y. Constantinides, "Fully coupled fluid-structure interaction for offshore applications," in International Conference on Offshore Mechanics and Arctic Engineering, 2009, vol. 43451, pp. 757-765 .

[7] M. R. Ross, M. A. Sprague, C. A. Felippa, and K. Park, "Treatment of acoustic fluid–structure interaction by localized Lagrange multipliers and comparison to alternative interface-coupling methods," Computer Methods in Applied Mechanics and Engineering, vol. 198, no. 9-12, pp. 986-1005, 2009.

[8] S. Turek and J. Hron, Proposal for numerical benchmarking of fluid-structure interaction between an elastic object and laminar incompressible flow. Springer, 2006.

[9] L. Shang, C. Hoareau, and A. Zilian, "Modeling and simulation of thin-walled piezoelectric energy harvesters immersed in flow using monolithic fluid–structure interaction," Finite Elements in Analysis and Design, vol. 206, p. 103761, 2022.

[10] J. Hron, A. Ouazzi, and S. Turek, "A computational comparison of two FEM solvers for nonlinear incompressible flow," in Challenges in Scientific Computing-CISC 2002: Proceedings of the Conference Challenges in Scientific Computing Berlin, October 2–5, 2002, 2003: Springer, pp. 87-109 .

[11] S. Bna, "Multilevel domain decomposition algorithms for monolithic fluid-structure interaction problems with application to haemodynamics," 2014.

[12] J. Degroote, K.-J. Bathe, and J. Vierendeels, "Performance of a new partitioned procedure versus a monolithic procedure in fluid–structure interaction," Computers & Structures, vol. 87, no. 11-12 ,pp. 793-801, 2009.

[13] P. Causin, J.-F. Gerbeau, and F. Nobile, "Added-mass effect in the design of partitioned algorithms for fluid–structure problems," Computer methods in applied mechanics and engineering, vol. 194, no. 42-44, pp. 4506-4527, 2005.

[14] E. H. van Brummelen, "Added mass effects of compressible and incompressible flows in fluid-structure interaction," 2009.

[15] S. R. Idelsohn, F. Del Pin, R. Rossi, and E. Oñate, "Fluid–structure interaction problems with strong added-mass effect," International journal for numerical methods in engineering, vol. 80, no. 10, pp. 1261-1294, 2009.

[16] U. Küttler, M. Gee, C. Förster, A. Comerford, and W. Wall, "Coupling strategies for biomedical fluid–structure interaction problems," International Journal for Numerical Methods in Biomedical Engineering, vol. 26, no. 3-4, pp. 305-321, 2010.

[17] H.-J. Bungartz, M. Mehl, and M. Schäfer, Fluid Structure Interaction II: Modelling, Simulation, Optimization. Springer Science & Business Media, 2010.

[18] U. Langer and H. Yang, "Numerical simulation of fluid-structure interaction problems with hyperelastic models I: A partitioned approach," arXiv preprint arXiv:1312.5561, 2013.

[19] G. A. Holzapfel, "Nonlinear solid mechanics: a continuum approach for engineering science," ed: Kluwer Academic Publishers Dordrecht, 2002.

[20] Z. Więckowski, "The material point method in large strain engineering problems," Computer methods in applied mechanics and engineering, vol. 193, no. 39-41, pp. 4417-4438, 2004.

[21] T. Wick, "Adaptive finite element simulation of fluid-structure interaction with application to heart-valve dynamics," 2011 .

[22] T. J. Hughes, W. K. Liu, and T. K. Zimmermann, "Lagrangian-Eulerian finite element formulation for incompressible viscous flows," Computer methods in applied mechanics and engineering, vol. 29, no. 3, pp. 329-349, 1981.

[23] J. Donea, S. Giuliani, and J.-P. Halleux, "An arbitrary Lagrangian-Eulerian finite element method for transient dynamic fluid-structure interactions," Computer methods in applied mechanics and engineering, vol. 33, no. 1-3, pp. 689-723, 1982.

[24] F. M. White and J. Majdalani, Viscous fluid flow. McGraw-Hill New York, 2006.

[25] A. E. Bogaers, S. Kok, B. D. Reddy, and T. Franz, "An evaluation of quasi-Newton methods for application to FSI problems involving free surface flow and solid body contact," Computers & Structures, vol. 173, pp. 71-83, 2016.