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Investigation of internal resonance of a beam with a local bi-stable nonlinear energy sink

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Abstract

This paper studies the nonlinear oscillations of a beam with simply supported boundary conditions to which a bi-stable NES (nonlinear energy sink) is locally attached.. Bi-stable NESs are used because they are more efficient than the mono-stable ones. The dimensionless the equations of motion of the system are derived, and by using Galerkin's method the equations of motion are discretized. Then, the method of multiple scales (MMS) is employed to find the 3:1 primary internal resonance has been investigated. The corresponding results show that the frequency responses are highly affected by slight changes in the NES parameters.

Keywords: Targeted energy transfer, The Galerkin method, Bi-stable nonlinear energy sink, Internal resonance.

1. Introduction

Vibration mitigation of large-scaled structures has consistently attracted the attentions of researchers. Vibrations can sometimes threaten the safety of structures or cause collapse and destruction by imposing stresses exceeding the safety factor. Bridges are one of the well-known large structures under various loads, some of which are periodic, like aerodynamic forces induced by surrounding air stream during storms. In this context, conducting an analysis is advantageous due to its potential for saving time and energy. Since such structures use energy dissipation methods such as NES to reach this aim, the focus of this paper is decided to be on analyzing the behavior of bridges with local bi-stable NES for the first time. A bi-stable NES takes the advantage of bi-stable springs to increase the performance of NES. An exhaustive literature review of the above mentioned introduction written in the main manuscript can be found in the references section [1-29].

2. Methodology

This section provides an overview of the paper's methodology. First, the governing equations of motion and their dimensionless forms for a bridge model (a simply supported continuous Euler beam as in Fig. 1) are derived. The dimensionless equations are presented in Eq. 1 as follows:

$$\begin{aligned}&\eta_{\tau\tau}+\eta_{\xi\xi\xi\xi}=F_1\\&\qquad\qquad+F_{NES}\delta_d(\xi-\xi_{NES})\\&\alpha_1\theta_{\tau\tau}-\alpha_2\theta+\alpha_3\theta^3=-F_{NES}\\&F_{NES}=-\beta_1\big(\theta-\eta(\xi_{NES},\tau)\big)+\\&\beta_2\big(\theta-\eta(\xi_{NES},\tau)\big)^3+\beta_3\big(\theta_\tau-\\&\eta_\tau(\xi_{NES},\tau)\big)\end{aligned} \tag{1}$$

Eq. 1 is discretized using the Galerkin method with the following mode shapes:

$$\begin{aligned}&\phi_n(\xi)=sin(n\pi\xi)\\&\qquad n\in N\end{aligned} \tag{2}$$

To analyse the internal resonance case: $\Omega\simeq\omega_1$ and $\omega_2\simeq3\omega_1$, the MMS is applied on the single-mode discretized equation of motion of the beam together with the equation of motion of the bi-stable NES. Studying this physical case of resonance emanates from the fact that the most vibration mitigation occurs whenever the NES absorbs the most possible energy of the system.

After rescaling the coefficients of Eq. 1 and applying a two-term solution to it, one can find the steady-state solution of Eq. 1 standing for the nonlinear frequency response of the system as follows:

$$\begin{aligned}&\frac{\pi^2\left(a_1^2\beta_3\omega_1+3a_2^2\beta_3\omega_2\right)^2}{4a_1^2f_0^2}+\\&\left(\frac{\pi^2}{64a_1^2f_0^2}\right)(\Gamma)^2=1\end{aligned} \tag{3}$$

The detail of parameters of derivations are discussed in the main manuscript.

3. Discussion and results

In this study, MMS is used to study the internal resonance of the bridge and the attached bi-stable NES. However, it is insufficient on its own. Thus, a numerical Runge-Kutta method is applied to Eq. 1 to find the numerical frequency response of the bridge. The result, illustrated in section 4 shows that both numerical and MMS frequency response curves conform well to each other, as can be seen in Fig. 2. Therefore, the remaining results are also validated. A key finding of this paper is that the beam (bridge) has a hardening and the NES has a softening behavior. This, implies that the NES designers should find an applicable optimum set of NES parameters to maximize the energy transfer under realistic constraints. An example is shown in Fig. 3 of the next section.

4. Tables and Figures

As mentioned in the previous sections, Figs. 1-3 are illustrated below.

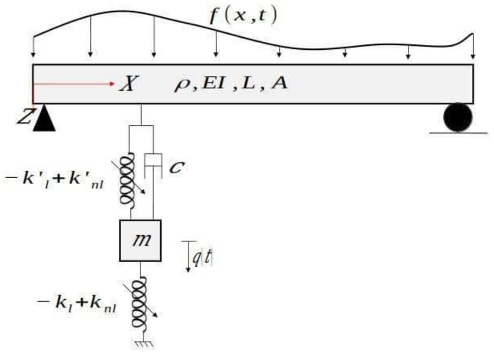


Figure 1. a bridge modeled as an Euler beam with bi-stable local NES

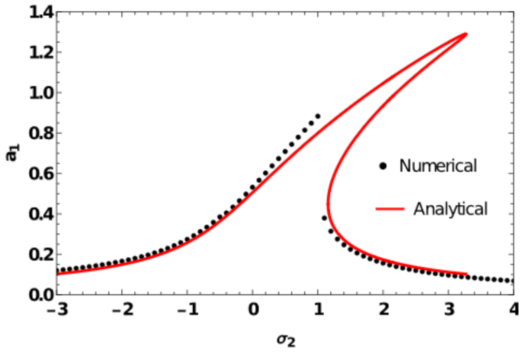
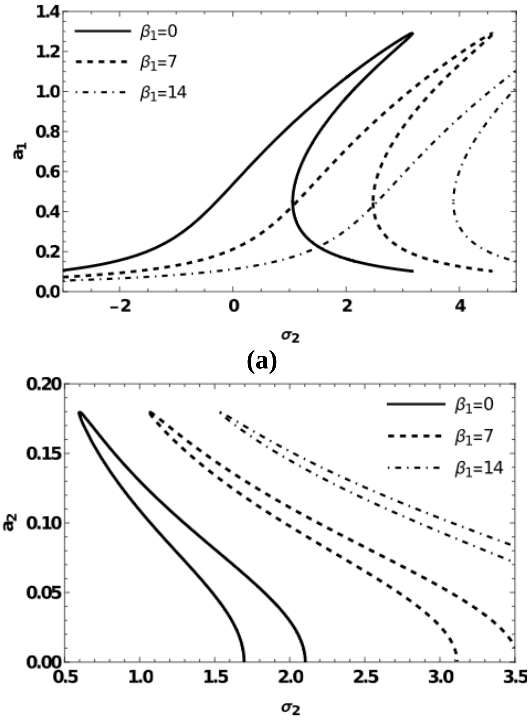


Figure 2. verifying the MMS frequency response of the beam with the Runge-kutta numerical method



(a)

(b)

Figure 3. the beam and the NES frequency responses, respectively shown in section (a) and (b)

5. System of Units

The entire formulation in this paper is dimensionless. Thus, one can extend the results to any system of units.

6. Conclusions

In this research, a large-scaled bridge-like structure is modeled as a simply supported continuous beam with a local bi-stable NES attached at a specific distance from the origin. The equations of motion of the coupled beam-NES system are derived using Newton’s second law of motion and these equations are then rendered dimensionless and discretized Galerkin’s method and comparing mode shapes of the beam. Taking the method of multiple scales into account, the modulation equation (frequency response) of the system is derived for the primary 3:1 internal resonance case and validated by the one obtained by a Runge-Kutta method. Finally, the effects of the NES dimensionless parameters on the beam’s frequency response are extensively studied and discussed.

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