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Free vibration analysis of a folded auxetic plate using the Levy-differential quadrature method

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Abstract

In this study, the vibrations of a folded plate made of auxetic cells were examined. Initially, the elastic constants and density of the auxetic plate were determined based on the geometrical and material parameters of unit cell. The folded plate was considered as two jointed rectangular plates. Utilizing the first-order shear deformation theory and applying Hamilton's principle, the equations of motion governing each plate and the boundary conditions at the plate's edges were derived. Next, employing the combined Levy-differential quadrature method, the transformed equations of motion from the partial type to ordinary one has been solved. Assembling the equations of motion with boundary and continuity relations leads to an eigenvalue problem that its solution can present the frequency response function of folded plate. To validate the results obtained from the Levy-differential quadrature solution, the auxetic plate was simulated by a finite element analysis software, and the comparison results demonstrated the accuracy of presented method. Finally, the effects of plate's geometrical parameters on the natural frequencies of the folded plate were investigated.

Keywords: Folded Plate; Auxetic; Free Vibration; Differential Quadrature Method; Finite Element Method

1. Introduction

In recent years, the use of auxetic materials has received attention due to their strength-to-weight ratio and high energy absorption. In 1987, Lakes developed foams with negative Poisson's ratio that exist in nature or can be manufactured [i]. In another study, the vibration of a folded plate of functionally graded composite material reinforced with graphene was investigated by numerical solution of differential quadrature method [ii]. Also, in another research, the dynamic behavior of flat and folded plates in a thermal environment was investigated using the theory of non-polynomial shear deformation [iii]. The theory of high-order shear deformation of the plate was used to analyze the transient dynamics of folded composite plates under different loading [iv]. The free vibration of a plate with three distinct parts, where one part is connected at two edges perpendicular to the other two parts, was studied by Zhang et al [v]. According to a review of research literature and the application of auxetic structures in various industries, the vibration analysis of plates consisting of folded auxetic cells can be very important, which has not been studied so far. Also, the use of the combined differential quadrature method, which is faster, more accurate and easier to use than the numerical methods used, is one of the other innovations of the present study. In this research, the equations of motion and boundary conditions of a plate consisting of folded auxetic cells are obtained by using the first-order shear deformation theory and using Hamilton's principle. Using the Levy-difference-of-squares method (Levy-DQM), the governing differential equations are solved and the results are compared with Abaqus software.

2. The geometry of the folded auxetic plate

Figure 1 shows a schematic of an auxetic unit cell.

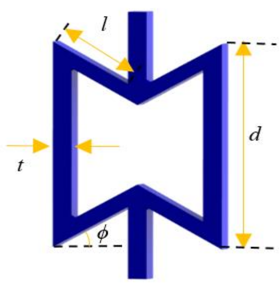

Figure 1. Schematic of the auxetic unit cell

Auxetic folded plate is shown in figure 2.

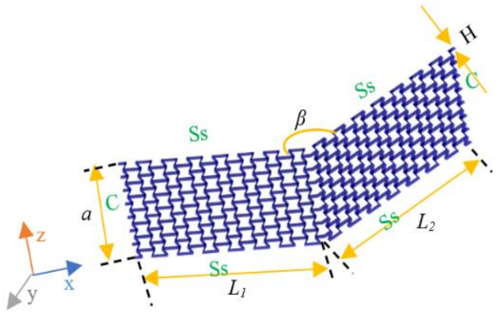

Figure 2. A schematic of a folded plate consisting of auxetic cells

3. Governing dynamic equations

Using the first-order shear deformation theory (FSDT), the displacement-strain and stress-strain field relations are expressed as relations (1).

$$\begin{bmatrix}\epsilon_{x_kx_k}\\ \epsilon_{y_ky_k}\\ \gamma_{x_ky_k}\end{bmatrix} = \begin{bmatrix}\partial u_k/\partial x_k\\ \partial v_k/\partial y_k\\ \partial u_k/\partial y_k + \partial v_k/\partial x_k\end{bmatrix} + z\begin{bmatrix}\partial \theta_{xk}/\partial x_k\\ \partial \theta_{yk}/\partial y_k\\ \partial \theta_{xk}/\partial y_k + \partial \theta_{yk}/\partial x_k\end{bmatrix}, \quad (1)$$

$$\begin{bmatrix}\gamma_{x_kz_k}\\ \gamma_{y_kz_k}\end{bmatrix} = \begin{bmatrix}\theta_{xk} + \partial w_k/\partial x_k\\ \theta_{yk} + \partial w_k/\partial y_k\end{bmatrix},$$

$$\begin{bmatrix}\sigma_{x_kx_k}\\ \sigma_{y_ky_k}\\ \sigma_{x_ky_k}\end{bmatrix} = \begin{bmatrix}Q_{k11} & Q_{k12} & 0\\ Q_{k21} & Q_{k22} & 0\\ 0 & 0 & Q_{k33}\end{bmatrix}\begin{bmatrix}\epsilon_{x_kx_k}\\ \epsilon_{y_ky_k}\\ \epsilon_{x_ky_k}\end{bmatrix},$$

$$\begin{bmatrix}\sigma_{x_kz_k}\\ \sigma_{y_kz_k}\end{bmatrix} = \begin{bmatrix}Q_{k44} & 0\\ 0 & Q_{k55}\end{bmatrix}\begin{bmatrix}\epsilon_{x_kz_k}\\ \epsilon_{y_kz_k}\end{bmatrix},$$

k=1, 2.

Where (u_k ‹v_k ‹w_k‹θ_x& θ_y) represent the components of displacement and rotation field in FSDT. The coefficients of the stiffness matrix for the auxetic structures are calculated according to the FSDT according to equation (2).

$$Q_{k11} = \frac{E_1}{1-\mu_{12}\mu_{21}},$$

$$Q_{k12} = \frac{\mu_{12}E_2}{1-\mu_{12}\mu_{21}}, Q_{k22} = \frac{E_2}{1-\mu_{12}\mu_{21}}, \quad (2)$$

$$Q_{k33} = G_{12}, Q_{k44} = G_{23}, Q_{k55} = G_{13}$$

Using Hamilton's principle, it is calculated according to the equations governing the plate and the boundary conditions at the edges.

$$\delta\int_0^t (U_k - T_k)\,dt = 0 \quad (3)$$

Five dynamic governing equations are obtained for each part of the folded plate according to relations (4).

$$\frac{\partial N_{x_kx_k}}{\partial x} + \frac{\partial N_{x_ky_k}}{\partial y} = J_{k1}\frac{\partial^2 u_k}{\partial t^2} + J_{k2}\frac{\partial^2 \theta_{xk}}{\partial t^2},$$

$$\frac{\partial N_{y_ky_k}}{\partial y} + \frac{\partial N_{x_ky_k}}{\partial x} = J_{k1}\frac{\partial^2 v_k}{\partial t^2} + J_{k2}\frac{\partial^2 \theta_{yk}}{\partial t^2},$$

$$\frac{\partial M_{x_kx_k}}{\partial x} + \frac{\partial M_{x_ky_k}}{\partial y} - Q_{x_k} = J_{k2}\frac{\partial^2 u_k}{\partial t^2} + J_{k3}\frac{\partial^2 \theta_{xk}}{\partial t^2}, \quad (4)$$

$$\frac{\partial M_{y_ky_k}}{\partial y} + \frac{\partial M_{x_ky_k}}{\partial x} - Q_{y_k} = J_{k2}\frac{\partial^2 v_k}{\partial t^2} + J_{k3}\frac{\partial^2 \theta_{yk}}{\partial t^2},$$

$$\frac{\partial Q_{xk}}{\partial x} + \frac{\partial Q_{yk}}{\partial y} = J_{k1}\frac{\partial^2 w_k}{\partial t^2},$$

The boundary conditions at the edges of the plate are according to the following relations.

Boundary conditions of the simple support at the edges along the $x_k$axis

$$u_k = v_k = w_k = \theta_{xk} = M_{y_ky_k} = 0 \quad (5)$$

clamped boundary conditions at the edges along the axis of y_k

$$u_k = v_k = w_k = \theta_{xk} = \theta_{yk} = 0 \quad (6)$$

Boundary conditions of the simple support at the edges along the $y_k$axis

$$u_k = v_k = w_k = M_{x_kx_k} = \theta_{y_k} = 0 \quad (7)$$

Free boundary conditions at the edges along the y_k axis

$$N_{x_kx_k} = N_{x_ky_k} = M_{x_kx_k} = M_{x_ky_k} = Q_{x_k} \quad (8)$$

At the connection boundary between two plates, there are continuity conditions of displacement, torsion, force and moment. The boundary conditions at the edge of the connection of two plates can be considered as relation (9).

Continuity of displacements and rotations:

$$u_1(L_1, y) = -u_2(0, y) \times \cos\beta - w_2(0, y) \times \sin\beta$$

$$v_1(L_1, y) = v_2(0, y) \quad (9)$$

$$w_1(L_1, y) = u_2(0, y) \times \sin\beta - w_2(0, y) \times \cos\beta$$

$$\theta_{x1}(L_1, y) = \theta_{x2}(0, y)$$

$$\theta_{y1}(L_1, y) = \theta_{y2}(0, y)$$

Continuity of forces and momentums:

$$\begin{aligned} -N_{x_1x_1}(L_1, y) \times \cos\,\beta + Q_{x_1}(L_1, y) \\ \times \sin\,\beta \\ = N_{x_2x_2}(0, y) \\ N_{x_1y_1}(L_1, y) = N_{x_2y_1}(0, y) \\ -N_{x_1x_1}(L_1, y) \times \sin\beta - Q_{x_1}(L_1, y) \\ \times \cos\beta = Q_{x_2}(0, y) \\ M_{x_1x_1}(L_1, y) = M_{x_2x_2}(0, y) \\ M_{x_1y_1}(L_1, y) = M_{x_2y_2}(0, y) \end{aligned} \qquad (10)$$

The differential quadrature method is used to solve the resulting equations.

4. Discussion and results

The geometrical parameters of the folded plate consisting of auxetic cells as well as the boundary conditions can be effective in the vibration frequency response.

Table 1 shows the effect of changing the boundary conditions on the natural frequency.

Table 1. The first four natural frequencies (Hz) of the auxetic folded plate through different boundary conditions

(N, M)	*Ss-Ss-Ss-Ss-Ss-Ss*	*F-Ss-Ss-F-Ss-Ss*
(1,1)	5.950	1.226
(1,2)	14.485	4.814
(1,3)	18.819	14.349
(1,4)	31.769	21.115

The first four natural frequencies of the auxetic folded plate for different ϕ angles are shown in Figure 3.

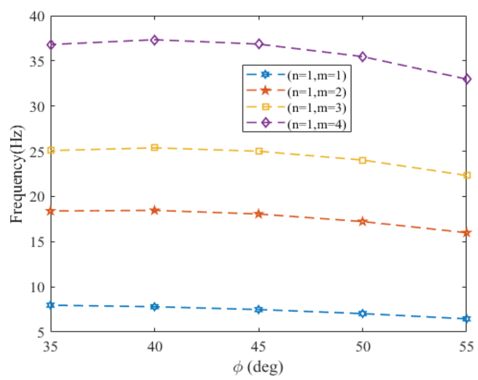

Figure 3. Four first natural frequencies for different levels

In Figure 4, the effect of changes in the plate thickness (H) on the vibrations of the auxetic folded plate is investigated.

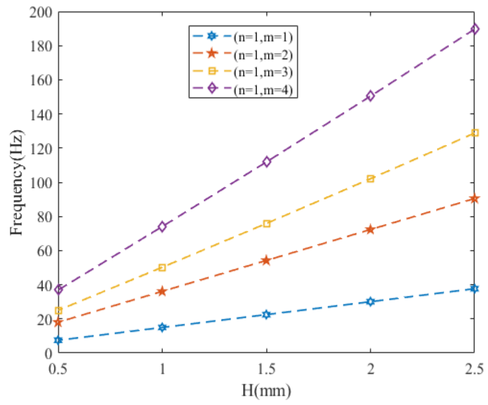

Figure 4. The first four natural frequencies for different plate thicknesses

The first four natural frequencies for different t are shown in Figure 5.

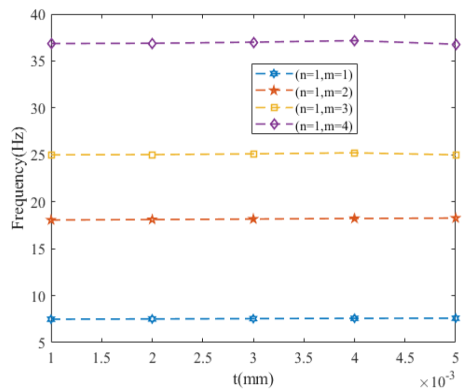

Figure 5. Four first natural frequencies for different t

5. Conclusions

In the present study, the vibrations of the folded plate consisting of auxetic cells were investigated using the Levy-differential quadrature method. The results of the solution were compared with the results of the finite element analysis, which shows the acceptable accuracy of the present Levy- differential quadrature method. Some results are as follows:

1. As the plate width increases and the rest of the parameters remain constant, the natural frequency decreases.
2. By reducing the angle β from 180 to the smaller and the plate changes from a flat state to a folded sheet, the natural frequencies increase very quickly and then

remain constant.

3. As the plate thickness increases, the natural frequencies increase.
4. As the K ratio increases, the natural frequencies decrease in the first two modes and increase and then decrease in the third and fourth modes.
5. The effect of increasing the thickness of the beams constituent the auxetic cell on the natural frequency is almost negligible.

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