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Comparison of nonclassical controllers on piezoelectric nanoresonator: natural frequency and pull in voltage analysis

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Abstract

In current work, some nonclassical controller effects such as strain gradient (SGT), nonlocal (NLT) and Gurtin–Murdoch surface/interface (GMSIT) theories are presented for analyzing of nonlinear vibration in piezoelectric nanoresonator (PENR) compared to classical theory (CT). PENR subjected to nonlinear electrostatic excitation with direct (DC) and alternating (AC) voltages and also visco-pasternak medium. For this analysis, Hamilton's principle, Galerkin technique, combination of Complex averaging method and arc-length continuation are used. The results show that ignoring small-scale and surface/interface effects give inaccurate predictions of vibrational response of the PENR. It is indicated that in different boundary condition, material length scale and nonlocal scale parameters respectively lead to decreasing and increasing of PENR stiffness and also the amplitude of oscillation and the range of instability of non-classic theories of NLT and SGT are greater than that of the classical one.

Keywords: Piezoelectric nanoresonator, Nonlocal strain gradient theory, Gurtin–Murdoch surface/interface theory, Natural frequency; Pull in voltage.

1. Introduction

Nanostructures are widely used in various fields of science and technology due to their small mass and size [1]. One of the most important nanostructures is a nanoresonator [2]. For dynamic analysis and mathematical modeling of these nanostructures, non-classical theories such as non-local theory [3], strain gradient [4] and Gurtin-Murdoch surface/interface theories [5] have been presented to investigate nonlinear vibrations and dynamic analysis of nanostructures.

Based on the theory of nonlocal elasticity, Najafi et al. [6] have analyzed the free vibrations of piezoelectric nanobeams and investigated the effects of different parameters such as nonlocal parameter, length to thickness ratio, and external applied voltage. The free vibration analysis of functional graded conical panels reinforced with graphene nanoplates has been investigated by Mirzaei et al [7]. By using the nonlocal strain gradient theory and the analytical method of multiple time scales, the nonlinear vibrations of the nonlocal Euler-Bernoulli nanowire have been investigated by Karamad et al. [8]. According to Gurtin-Murdoch theory, free vibration of nano-sized piezoelectric double-shell structures have been studied by Fang et al [9]. Recently, Hashemi Kachapi et al. some important analytical methods on a small scale, such as Gurtin–Murdoch surface/interface energy theories, Eringen's non-local theory and non-local strain gradient, as well as the combination of these different methods. have presented to investigate the effects of non-classical controllers on natural frequencies, nonlinear vibrations and stability analysis of multi-walled piezoelectric nanostructures under various excitations such as harmonic, visco-pasternak and nonlinear electrostatics [10-14]. The behavior of the Kirchhoff nanoplate is studied by Sheikhlo et al., using the Couple stress theory and the Gurtin-Murdoch surface theory [15].

It should be noted that a very limited number of studies simultaneously studied the effect of surface/interface energy and small-scale effects for nanostructures, especially piezoelectric nanostructures. Yiyuan et al. investigated the surface-mass interactions by using the non-local surface method and its application in the mechanics of nanobeams [16]. Ghorbanpour et al. [17] have analyzed the nonlinear vibrations of single and double-walled nanosheets using non-local piezoelasticity and surface energy theories.

In all the previous works that have been done by the authors, very few researches have been done in the analysis of vibrations and stability of piezoelectric nanostructures by simultaneously considering the effects of strain gradient, Gurtin–Murdoch surface/interface and non-local effects. The current research is a continuation of the work done [13], but the subject of study, especially the actuation forces, is completely different from the previous work, and as a result, different results are obtained from the previous article, and unlike the previous work, the current nanostructure is simultaneously subjected to nonlinear electrostatic stimulation with direct (DC) and alternating (AC) voltages, as well as Visco Pasternak medium.

2. Mathematical formulation and Solution procedure

A piezoelectric nanoresonator based on cylindrical nanoshell subjected to visco-pasternak medium and nonlinear electrostatic excitation is shown in Figure 1. All of the physical and geometrical properties of the mentioned nanostructures can be seen in reference Hashemi Kachapi et al. [11].

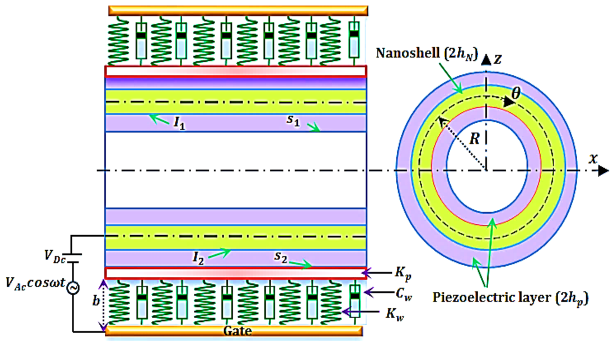

Figure 1. A piezoelectric nanoresonator subjected to nonlinear electrostatic excitation

The governing equations of motion and corresponding boundary conditions of the piezoelectric shell are obtained by applying the following Hamilton principle:

$$\int_0^t (\delta T - \delta\pi + \delta w_{vf} + \delta w_e)dt = 0, \tag{1}$$

For this purpose, the total strain energy of PENS considering the surface/interface effect can be presented as:

$$\pi = \frac{1}{2}\int_0^L \int_0^{2\pi} \left\{ \begin{array}{c} N_{xx}\varepsilon_{xx}^0 + N_{\theta\theta}\varepsilon_{\theta\theta}^0 \\ +N_{x\theta}\gamma_{x\theta}^0 + M_{xx}\kappa_{xx} \\ +M_{\theta\theta}\kappa_{\theta\theta} + M_{x\theta}\kappa_{x\theta} \\ +\eta_{33}\bar{E}_{zp}^2 h_p \end{array} \right\} Rd\theta dx \tag{2}$$

In Eq. (2), the forces (N) and moment (M) resultants are determined in [11, 13]. The first variation of kinetic energy of the PENS can be written as:

$$\delta\int_{t_1}^{t_2} Tdt = -\int_{t_1}^{t_2}\iint\left\{I\left(\left(\frac{\partial^2 u}{\partial t^2}\right)\delta u + \left(\frac{\partial^2 v}{\partial t^2}\right)\delta v + \left(\frac{\partial^2 w}{\partial t^2}\right)\delta w\right)\right\}Rd\theta dxdt \tag{3}$$

where

$$\begin{aligned} I &= \int_{-h_N}^{h_N}\rho_N\, dz + \int_{-h_N-h_p}^{-h_N}\rho_p\, dz \\ &+ \int_{h_N}^{h_N+h_p}\rho_p\, dz + \rho^{S,I} \\ &= 2\rho_N h_N + 2\rho_p h_p + 2\rho^S + 2\rho^I \end{aligned} \tag{4}$$

Also, and first variation of the work done by the viscoelastic foundation and nonlinear electrostatic excitation, respectively, can be written as [11, 13]:

$$\delta W_{vf} = -\int_0^L\int_0^{2\pi}\int_0^w \left(\begin{array}{c} K_w w \\ -K_p\nabla^2 w \\ +C_w\dfrac{\partial w}{\partial t}\end{array}\right)\delta w\, Rd\theta dx, \tag{5}$$

$$\delta W_e = \int_0^L\int_0^{2\pi}\int_0^w \frac{\pi\Upsilon(V_{DC}+V_{AC}cos(\omega t))^2}{\left(\begin{array}{c}\sqrt{\begin{array}{c}(b-w)\\ \times(2R+b-w)\end{array}} \\ \times\left[\cosh^{-1}\left(\dfrac{1+}{\dfrac{b-w}{R}}\right)\right]^2\end{array}\right)}\delta w\, Rd\theta dx \tag{6}$$

It is important to note that all relationships, coefficients and phrases for nonlocal strain gradient, surface/interface theories and nonlocal strain gradient surface/interface and small-scale stress-strain relationships and etc. can be seen in full detail in [11, 13]. In current study with expressing the electrostatic force Equation (6) as a polynomial form that as nonlinear curve-fitting problem is solved by lsqcurvefit function in Matlab Toolbox using least-squares, the dimensionless work done by electrostatic force can be express as follows [13]:

$$\delta W_e = \int_0^L\int_0^{2\pi}\left\{\int_0^{\bar{w}}\bar{F}_e(\bar{V}_{DC} + \bar{V}_{AC}\cos(\Omega\tau))^2(\bar{C}_1 + \bar{C}_2\bar{w} + \bar{C}_3\bar{w}^2 + \cdots + \bar{C}_n\bar{w}^{n-1})\,\delta\bar{w}\right\}\delta\theta\delta\xi \tag{7}$$

By applying the Galerkin method, Hamilton's principle, least-squares polynomial format of electrostatic force, using the displacement and shear deformation in the assumed mode method, applying non-dimensional strain and kinetic energies and also non-dimensional works and then substituting in the Lagrange-Euler equations, the dimensionless governing equations of motion and boundary conditions for PENS are obtained to the following equations:

$$\begin{aligned}&[(K)_u^u + (K_{bc})_u^u]\{\bar{u}\} + [(K)_u^v + (K_{bc})_u^v]\{\bar{v}\}\\ &\quad + [(K)_u^w + (K_{bc})_u^w]\{\bar{w}\}\\ &\quad + [(NL)_u^w + (NL_{bc})_u^w]\{\bar{w}^2\}\\ &\quad = [(M)_u^u]\{\ddot{\bar{u}}\} + \bar{F}_{up}^{bc},\end{aligned} \tag{8}$$

$$\begin{aligned}&[(K)_v^u + (K_{bc})_v^u]\{\bar{u}\} + [(K)_v^v + (K_{bc})_v^v]\{v\}\\ &\quad + [(K)_v^w + (K_{bc})_v^w]\{\bar{w}\}\\ &\quad + [(NL)_v^w + (NL_{bc})_v^w]\{\bar{w}^2\}\\ &\quad = [(M)_v^v]\{\ddot{\bar{v}}\} + \bar{F}_{vp}^{bc},\end{aligned} \tag{9}$$

$$\begin{aligned}&[(K)_w^u]\{\bar{u}\} + [(K)_w^v]\{\bar{v}\}\\ &+ \left[(K)_w^w + (K_{bc})_w^w - \left(K_{vp}\right)_w^w - (K_{e2})_w^w\right]\{\bar{w}\}\\ &+ [(NL)_w^u + (NL_{bc})_w^u]\{\bar{w}\bar{u}\}\\ &+ [(NL)_w^v + (NL_{bc})_w^v]\{\bar{w}\bar{v}\}\\ &+ [(NL)_{w2}^w + (NL_{bc})_{w2}^w - (NL_{2e})_{w2}^w]\{\bar{w}^2\}\\ &+ [(NL)_{w3}^w + (NL_{bc})_{w3}^w - (NL_{3e})_{w3}^w]\{\bar{w}^3\}\\ &= ([(M)_w^w + (M_{bc})_w^w])\{\ddot{\bar{w}}\} + ([(C)_w^w]\\ &+ [(C_{bc})_w^w])\{\dot{\bar{w}}\}) + \bar{F}_{wp} + \bar{F}_{wp}^{bc} - \bar{F}_{we}\\ &- \bar{F}_e \left\{\begin{matrix}((\bar{V}_{AC}\cos\bar{\omega}\tau)^2 + 2\bar{V}_{AC}\bar{V}_{DC}\cos\bar{\omega}\tau) \times \\ \begin{pmatrix}\bar{C}_4(NL_e)_{w3}^w + \bar{C}_3(NL_e)_{w2}^w \\ +\bar{C}_2(K_e)_w^w + \bar{C}_1\bar{F}_1\end{pmatrix}\end{matrix}\right\}\end{aligned} \tag{10}$$

All steps to obtain the governing equations and also all coefficients and phrases in equations (8)- (10) are defined in [13]. Verification, comparison and convergence study is investigated with full details for PENS in [11, 13].

3. Results and discussions

In this section, the effect of geometric parameters and different materials with and without strain gradient, non-local and surface/interface effects on the dimensionless natural frequency and pull-in instability analysis are presented. For this purpose, different boundary condition such as simply supported edge (SS) is presented. The surface and bulk material properties of Aluminum (Al) nanoshell and PZT piezoelectric layer, others physical and geometrical parameters of PENR and also two case of surface density are shown in Table 1 [11, 12, 17].

Table 1. Surface and bulk properties of Al and PZT-4 and other material and geometrical parameters

$\boldsymbol{E_N}$ $(\boldsymbol{GPa})$	υ_N	ρ_N $(\frac{kg}{m^3})$	λ^I (N/m)	μ^I (N/m)	τ_0^I (N/m)	ρ^I $(kg/m^2$
70	0.33	2700	3.786	1.95	0.9108	5.46×1

$\boldsymbol{C_{11p}}$ $(\boldsymbol{GPa})$	C_{22p} (GPa)	C_{12p} (GPa)	C_{21p} (GPa)	C_{66p} (GPa)	E_p (GPa)
139	139	77.8	77.8	30.5	95
$\boldsymbol{\upsilon_p}$	ρ_p $(kg\,m^{-3})$	η_{33p} $(10^{-8}$ $F/m)$	$\lambda^S(N/m)$	$\mu^S(N/m)$	τ_0^S (N/m)
0.3	7500	8.91	4.488	2.774	0.6048
$\boldsymbol{e_{31p}}$ $(\boldsymbol{C/m^2})$	e_{32p} (C/m^2)	e_{31p}^S (C/m)	e_{32p}^S (C/m)	$\rho^S(kg/m^2)$	
−5.2	−5.2	-3×10^{-8}	-3×10^{-8}	5.61×10^{-6}	

$\boldsymbol{R(m)}$	L/R	h_N/R	h_p/I	b/R	$C_w\left(\frac{N.S}{m}\right.$
$\boldsymbol{1 \times 10^{-9}}$	10	0.01	0.005	0.1	1×10^{-3}
$\boldsymbol{K_w(N/m^3)}$	$K_p(N/m)$	$V_p(V)$	V_0	$V_{DC}(V)$	$V_{AC}(V)$
$\boldsymbol{9 \times 10^{17}}$	2.07	1×10^{-5}	1	1.5	0.5
$\boldsymbol{\mu(m^2)}$	$\eta(m^2)$				
$\boldsymbol{(1 \times 10^{-10})^2}$	$(1 \times 10^{-11})^2$				

Case 1		Case 2	
$\boldsymbol{\rho^I(kg/m^2)}$	$\rho^S(kg/m^2)$	$\rho^I(kg/m^2)$	$\rho^S(kg/m^2)$
$\boldsymbol{5.46 \times 10^{-7}}$	5.61×10^{-6}	5.46×10^{-8}	5.61×10^{-7}

Figure 2 shows the comparison of three non-classical theories NLT, SGT and GMSIT with the classical CT theory on dimensionless natural frequency for the ratio of piezoelectric thickness to different radius h_p/R in SS PENR. From this figure, it can be seen that the highest frequency is related to GMSIT (density of case 2) and it shows that in this case, the rigidity of the system is higher than other cases. The lowest frequency also corresponds to the simultaneous consideration of GMSIT (case 1) and SGT ($\bar{\mu} = 0.1, \bar{\eta} = 0.01$), that is, GMSIT+SGT, and shows that in this case, the difficulty of the system is less than other cases. It is clear that the natural frequency of the classical theory is higher than the NLT and SGT theories and it indicates the reduction of system rigidity due to the consideration of these two theories. Also, due to greater rigidity, NLT theory has a higher frequency than SGT theory. Also, considering surface/interface effects and using higher or lower surface/interface density can create lower or higher frequencies than classical frequencies, but using both NLT and SGT frequency theory It will be less natural than the classic mode

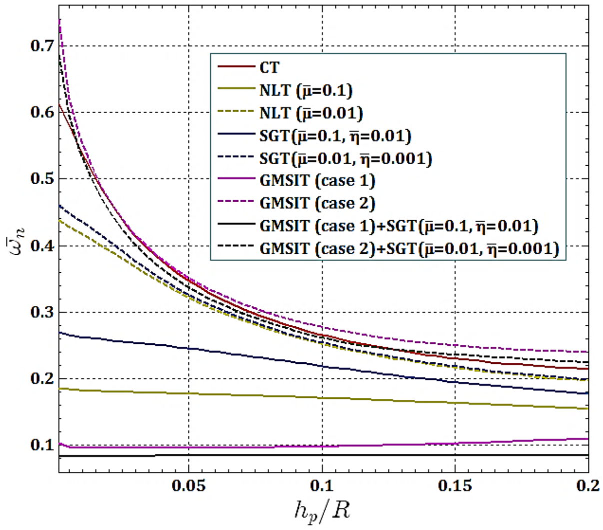

Figure 2. Comparison of non-classical theories with classical theory in dimensionless natural frequency versus piezoelectric thickness at different radius $\boldsymbol{h_p/R}$

The comparison of three non-classical theories NLT, SGT and GMSIT with classical CT theory for natural frequency versus direct DC voltage of SS nanoshell is presented in Figure 3. As it is known, considering the effects of surface/interface energy makes the system stiff and leads to the increase of DC voltage to reach the

Pauline voltages. Due to low stiffness in NLT, SGT and CT theories, they will reach the Pauline voltage sooner (and almost equal). Also, for zero natural frequency, SS PENR becomes unstable and this physically shows that first PENR loses its stability due to the divergence caused by pitchfork bifurcation.

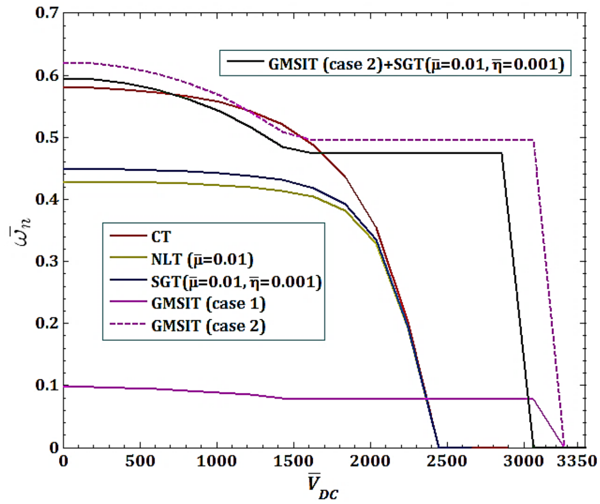

Figure 3. Comparison of non-classical theories with classical theory in dimensionless natural frequency versus DC direct Pull in voltage for SS PENR

4. Conclusions

In the present study, non-local, strain gradient and Gurtin-Murdoch surface/interface theories are presented to investigate dimensionless natural frequency analysis of piezoelectric nano-resonator under nonlinear electrostatic excitation and visco-pasternak medium in comparison with classical theory. For this analysis, Hamilton's principle and Galerkin method have been used to compare three non-classical theories NLT, SGT and GMSIT with the classical CT theory.

The results indicated that the natural frequency of the CT is greater than the NLT and SGT theories, indicating a reduction in the system rigidity due to the consideration of these two theories. Also, considering surface/interface effects and using higher or lower surface/interface density can create lower or higher frequencies than classical frequencies. Also, considering the effect of piezoelectric voltage on natural frequency, case 1 and 2 GMSIT as well as simultaneous consideration of GMSIT and SGT, especially in the case of low density, the most changes in natural frequency are observed. In these cases, as the voltage increases and as a result the system stiffness increases, the frequency increases. In other theories, changes in piezoelectric voltage do not have a significant effect on the natural frequency of PENR. Comparing the three non-classical theories NLT, SGT and GMSIT with the classical CT theory, considering the surface/interface effects makes the system stiff and leads to an increase in the DC voltage to reach the Pull in voltages. Due to the low stiffness in NLT, SGT and CT theories, they will reach the Pull in voltage sooner and for the zero natural frequency, SS PENR, they will lose their stability due to the divergence caused by pitchfork bifurcation.

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