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Quasi-3D Buckling Analysis of FG Sandwich Plates with Piezoelectric Face-Sheets Based on Refined High-Order Shear Deformation Theory

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Abstract

This research investigates the quasi-3D buckling behavior of the functionally graded plate integrated with piezoelectric face sheets as a fundamental and sensitive sandwich structure in various industries based on the refined high-order shear theory by considering the stretching effects along the thickness. The mechanical properties of the core are assumed to be non-homogeneous and the face sheets are modeled based on the theory of piezoelasticity. The equations of motion are extracted using the principle of virtual work based on axial and biaxial loading conditions. After the adaptation and evaluation of the present method, the critical buckling loads of the three-layer plate subjected to an external electric field are calculated in different conditions using Navier's analytical method. Finally, the effects of various parameters including volume fraction index, geometric dimensionless parameters, and comparison of two-dimensional and pseudo-three-dimensional models are examined. The results show that the difference between the 2D and quasi-3D models decreases with the width-to-length ratio increase and the thickness decrease. Eventually, the results show that the theory used in modeling is very accurate in addition to being remarkably simple.

Keywords: Axial and Biaxial Buckling, Quasi-3D, FG Core, Refined Theory.

1. Introduction

Multilayer structures are one of the structures that have gained significant attention in aerospace, marine, automotive, and transportation industries due to their high strength-to-weight ratio, stiffness, exceptional resistance, lightweight nature, relative insulation against heat and sound, as well as their manufacturability and stability. Functionally graded materials (FGM) are those whose volume fraction continuously and gradually changes from one surface to another in one or two directions. The physical and mechanical properties of these materials have made them relevant for use in aerospace, optoelectronics, and biomechanics. Based on these characteristics, many researchers have focused on studying these materials.

Hajlaoui et al. [1] analyzed the thermal buckling behavior of FG shells using a modified first-order solid shell element formulation. Hosseini-Hashemi et al. [2] studied the free vibrations of non-uniform and unidirectional FG Euler-Bernoulli beams with non-classical boundary conditions. Nguyen et al. [3] presented an analysis of thermal buckling and vibrations of FG plates using higher-order shear theory. Arefi and Soltan Arani [4] investigated the non-local free vibrations of three-layer FG nanoplates based on higher-order shear theory, considering the effects of stretching. Mohammadi et al. [5] examined the effect of incompressibility on the static, dynamic, and stability responses of thick FG plates. They utilized higher-order deformation theory to derive the equations of motion and found that the critical buckling mode depends on the loading conditions, aspect ratio, and material properties.

There are various theories for analyzing plates and the most widely used of them are the classical theory and shear deformation theory. The classical theory does not consider the effect of transverse shear deformation, and the accuracy of the answers of this theory decreases with the increase of the thickness of the plates. In first-order shear deformation theory (FSDT), the effects of transverse shear strains and also the effect of rotational inertia are taken into account. In this theory, a shear correction factor is required due to the non-zero shear stresses at the free surfaces of the plate. Higher-order shear deformation theories (HSDT) include third-order, and fifth-order theories, as well as harmonic, hyperbolic, parabolic, and exponential shear deformation theories (SDT), refined plate theory, etc. These theories do not require the use of a shear correction factor because they consider zero shear stress at the free surfaces of the plate. Daikh et al. [6] investigated the bending deflection and non-local stress distribution of FG sandwich nanosheets supported on a

Winkler elastic foundation based on the quasi-three-dimensional hyperbolic shear deformation theory. Karimi et al. [7] presented an analytical study on the static behavior of sandwich plates made of isotropic face sheets and a honeycomb core using HSDT. Mamandi and Mirzaei ghaleh [8] studied the nonlinear vibrations of an isotropic microbeam supported on a Winkler foundation under the influence of axial compressive load using the Euler-Bernoulli model, considering the modified coupling stress theory. The results indicate that as the axial load increases, the ratio of nonlinear to linear frequency increases. The buckling analysis of FG porous sandwich plates located on a Pasternak foundation by applying the new refined quasi-three-dimensional hyperbolic shear deformation theory is studied by Vu et al. [9]. Soltan Arani et al. [10] investigated the free vibration behavior of a non-local FG nanoplate supported on an orthotropic visco-Pasternak foundation by considering the refined quasi-3D HSDT and surface effects.

According to the comprehensive mentioned literature review survey and the best of the authors' knowledge, so far, there has been no attempt concerning investigate the uniaxial and biaxial static buckling behavior of FG sandwich plates integrated with piezoelectric layers by considering the thickness stretching effects. The motivation for this study is to provide an accurate solution for the quasi-3D buckling analysis of FG plates integrated with piezoelectric faces using refined shear deformation theory under the influence of an external electric potential. Moreover, understanding the buckling behavior of sandwich systems can be crucial for the application of electromechanical resonators. This study considers, for the first time, the stretching effects and HSDT to examine and compare the 2D and quasi-3D behaviors of FG sandwich plates. Ultimately, one of the innovations of this research is the presentation of comparative results in different models for the critical buckling load of the sandwich structure.

2. Fundamental Equations

A three-layer plate consisting of piezoelectric face sheets and a FG core under a uniform potential field with length a, width b, and total thickness h is demonstrated in Figure 1. The total thickness h includes the core thickness h_c and the thickness of the face sheets h_p. Generally, it is assumed that h_p is very small compared to τ_{13} and τ_{23}, except at the edges of the shell. There is complete continuity between the layers, and there are no sliding conditions between the faces and the core [9,10].

2.1. Functional Graded Materials

In this study, the core of the plate is considered to be made of FGMs so their mechanical properties vary continuously along the thickness based on the characteristics of a ceramic material and a metallic material. The effective properties of the materials are calculated using the mixture approach.

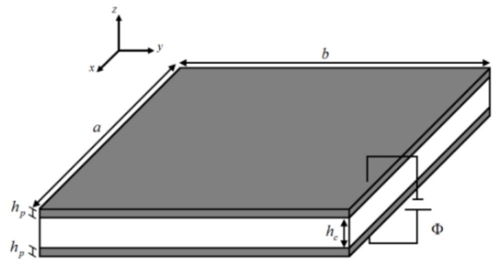

Figure 1. Schematic of the coordinate system and geometry of a three-layer sheet

Accordingly, the volume fraction of the materials and consequently the properties of the core are determined using a power function along the thickness [11,12].

$$V_C = (1/2 + z/h_c)^k \tag{1}$$
$$V_C + V_M = 1 \tag{2}$$

In (1), C and M are the subscripts that refer to ceramic and metal and also k denotes the gradient index that is attributed to the change in volume fraction of the material composition. The effective properties of the materials for the two materials, metal and ceramic, are presented in (3,4). In these relations, P represents the properties of the materials, and V is the volume fraction of the metal and ceramic.

$$P = P_C V_C + P_M V_M \tag{3}$$
$$P(z) = (P_C - P_M)(1/2 + z/h_c)^k + P_M \tag{4}$$

2.2. Displacement Field

The quasi-3D displacement field based on the assumptions of the refined plate theory is presented according to the following relations. Due to the static nature of the present problem, the dependence of the terms on time has been ignored [13]:

$$u(x,y,z) = u_0 - z\,(\partial w_b/\partial x) - f(z)\,(\partial w_s/\partial x) \tag{5}$$
$$v(x,y,z) = v_0 - z\,(\partial w_b/\partial y) - f(z)\,(\partial w_s/\partial y) \tag{6}$$
$$w(x,y,z) = w_b + w_s + g(z)\varphi \tag{7}$$

In the above equations, u_0 and v_0 represent the displacements in the mid-plane, while w_b and w_s are the bending and shear components of the transverse displacement, respectively. Additionally, the displacement φ is added to account for normal stress (stretching effect). The shape function $f(z)$ is a third-order model used here [10,14]. In this study, the components of the electric field are assumed based on the distribution of the electric potential as a combination of a linear function and a trigonometric function [9,10,15-18].

$$f(z) = z[(5/3)(z/h)^2 - 1/4] \tag{8}$$

$$g(z) = 1 - f'(z) \quad (9)$$

2.3. Equations of Motion

The governing equations of motion for this sandwich structure considering non-zero strains, structural relations, and electrical displacement are defined based on the theory of piezo-elasticity and Hooke's law by employing the principle of virtual work, according to (10). In this equation, W and U represents the work done by the external force and the strain energy, respectively. [9-12].

$$\int_0^t (\delta U - \delta W)dt = 0 \quad (10)$$

The variations in strain energy of the structure including the FG core and the piezoelectric face sheet are calculated using the principle of minimum potential energy [17].

$$\delta U = \iiint \left[\sigma_{ij}\delta\varepsilon_{ij} - D_i \delta E_i\right] dV \quad (11)$$

By substituting the strains and electric fields in the above equation and integrating along the thickness of the structure, the strain energy is obtained based on the force and moment terms. The external work changes can be presented based on (12).

$$\delta W = \iint (\bar{N})\delta w \, dA \quad (12)$$

Finally, by substituting (12) and (11) into (10) and separating and setting the coefficients $\delta u, \delta v, \delta w_b, \delta w_s, \delta\varphi$ and $\delta\phi$ to zero, the governing equations of the three-layer structure consisting of a FG core and piezoelectric face sheets under an electric field are obtained.

3. Solution Method, Numerical Results and Discussion

In this section, the analytical results for determining the critical buckling load of a three-layer structure composed of a FG core and piezoelectric layers are presented using the refined quasi-3D theory and the Navier solution method. The Navier method provides a powerful tool for analyzing structures and plates by utilizing simply supported boundary conditions and precise mathematics. Before presenting the results, to evaluate and validate the proposed theory, the comparison of the uniaxial and biaxial critical buckling load of the FG plates based on various width-to-thickness ratios has been examined by [17] in Figure 2. This Figure shows a good agreement between the existing results and those predicted using the current model.

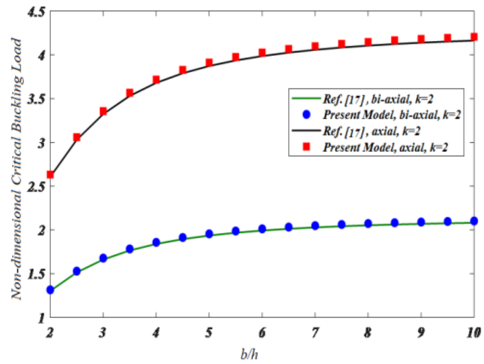

Figure 2. Comparison of dimensionless critical load in terms of width-to-thickness ratio

The non-dimensional critical load of the three-layer structure with a FG core and piezoelectric face sheets in terms of the aspect ratio of the structure is demonstrated in Figure 3. In this Figure, the critical load is examined for various thickness-to-length ratios of the structure versus the width-to-length ratio of the plate. To analyze the results in this section, assume that $(h_c/h_p) = 10$, $V_0 = 10$ and the power index value is equal to 2 ($k = 2$). By an increase in the width-to-length ratio of the structure which is caused by increasing the width or decreasing the length of the structure, the plate's surface area increases in both uniaxial and biaxial conditions.

A larger surface area implies greater flexibility and reduces in stiffness of the structure which consequently leads to a decrease in the non-dimensional critical buckling load. On the other hand, an increase in the thickness-to-length ratio of the plate due to the increase in the volume mass of the structure increases the critical load and enhances the stability of the structure. As can be observed from Figure 3, the effect of the thickness-to-length ratio on the plate is more pronounced at lower width-to-length ratios, which is clearly due to the reduction in the surface area of the structure and the increase in the volume mass of the structure.

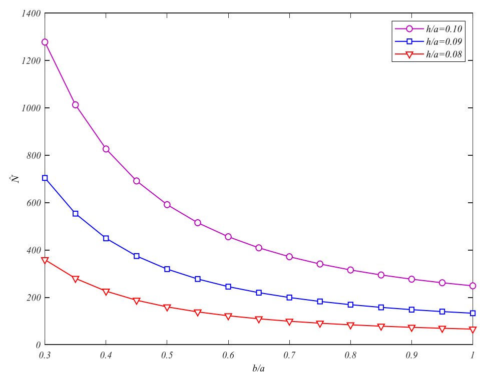

Figure 3. Variations of non-dimensional critical load versus a function of aspect ratio

A comparison of the critical buckling load in uniaxial and biaxial cases for the 2D and quasi-3D theories based on different width-to-length ratios is presented in Figure 4. In this case the ratio of thicknesses is considered equal to10 and also power low index is assumed to be 2 ($h_c/h_p = 10, V_0 = 10\,, k = 2$).

As can be seen, the quasi-3D theory provides a lower critical load compared to the 2D theory. Additionally, it is also observed that the quasi-3D theory has a greater effect on uniaxial buckling. Considering the refined quasi-3D theory leads to a 13.2% reduction in the critical load at ($b/\mathrm{a} = 0.3$) and a 16% reduction at ($b/\mathrm{a} = 0.6$). Based on the results of the 2D model and the comparison of results in Figure 4, it can be concluded that the use of quasi-3D theories and consideration of conditions close to the reality structure is essential and undeniable for a better understanding of structural behavior.

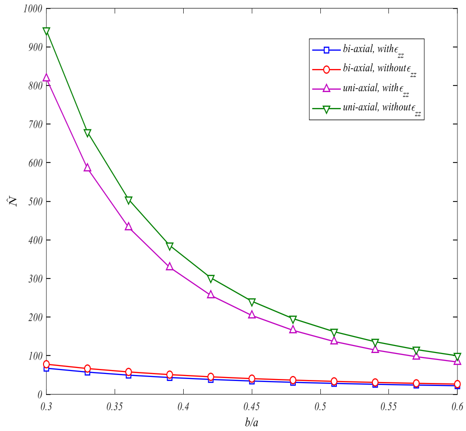

Figure 4. Comparison of dimensionless critical load in 2D and quasi-3D cases

4. Conclusions

In this study, the quasi-3D behavior of a sandwich plate with a FG core and piezoelectric face sheets is investigated using a refined plate theory. The FG core is considered a combination of metal and ceramic, and the modeling of equations for the piezoelectric layers is performed using piezo-elasticity theory. The governing equations are derived based on the principle of virtual work. The results indicate that considering the quasi-3D model leads to increased accuracy in buckling results. Since the influence of the quasi-3D approach is applied by employing the stretching effects along the thickness, the increase in the thickness of the core and face sheets has a significant impact on the buckling behavior of the plate. Additionally, it can be observed from the results that the influence of the thickness-to-length ratio and power index decreases with an increasing aspect ratio of the plate. From the comparison of uniaxial and biaxial buckling results, it can be concluded that the critical load in the biaxial condition is about 20% higher than the critical buckling load in the uniaxial condition.

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