Journal of Solid and Fluid Mechanics (JSFM), 14(4): 1-4, 2024

Shahrood University of Technology

Journal of Solid and Fluid Mechanics (JSFM)

DOI: 10.22044/jsfm.2024.14587.3861

Manufacturing Engineering of Iran

Employing the Isogeometric method for analyzing the in-plane and out-of-plane vibrations of laminated composite beams

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Received: 07/19/2024 Revised: 09/13/2024 Accepted: 10/26/2024

Abstract

This research mainly presents the in-plane and out-of-plane vibration analysis of composite beams with arbitrary lay-ups using isogeometric method. In this study, the first-order shear deformation theory and NURBS basis functions are used to obtain the free vibration response of the structure. The material couplings, i.e., bending-stretching, bending-twisting, and stretching-twisting couplings, and the effects of shear deformation, rotary inertia and Poisson's effect are considered. The obtained results using the isogeometric approach show excellent agreement with the results available in the open literature. The convergence study has been done using three different refinement schemes such as h-, p-, and k-refinement. It is observed that p-refinement has a faster convergence than h-refinement. Additionally, k-refinement is more suitable than p-refinement due to the lower number of degrees of freedom. It can also be concluded that using isogeometric analysis, the frequencies converged rapidly compared to the finite element method. Finally, the effects of various parameters on in-plane bending, out-of-plane bending, axial, and torsional vibration modes are investigated.

Keywords: Isogeometric analysis; NURBS functions; Laminated composite beam; Out-of-plane vibration; In-plane vibration.

1. Introduction

Composite structures are increasingly vital in engineering due to their advantageous properties such as high strength, low weight, and corrosion resistance. Understanding their dynamic characteristics is essential for practical applications. These structures are typically analyzed using analytical, semi-analytical, or numerical methods. Among recent advancements, the IsoGeometric Analysis (IGA) has emerged to address the limitations of traditional approaches.

Numerous studies have explored the behavior of composite beams using various methods. Analytical methods, such as those by Krishnaswamy et al. [1] and Friedman & Abramovich [2], have focused on vibrational and buckling characteristics, often incorporating effects like transverse shear deformation and rotary inertia. Other researchers, including Jafari-Talookolaei [3] and Aydogdu [4], extended these analyses to rotating beams and thermal buckling using first-order shear deformation theory and semi-analytical methods.

Some studies have used the finite element method, as seen in the work of Chandrashekhara and Bangera [5], but out-of-plane vibrations in laminated composite beams with arbitrary layering have received less attention. Yıldırım and Kıral [6] and Çalım [7] have partially addressed this gap, while Jafari-Talookolaei et al. [8] further highlighted the significant errors introduced by neglecting out-of-plane displacements.

The isogeometric method has been used in structural analysis to achieve higher accuracy, especially for complex geometries. Researchers like Lee and Park [9] and Nguyen et al. [10] have applied it to study Timoshenko beams and alleviated issues like shear locking through advanced refinement techniques. Despite these advances, limited attention has been given to the out-of-plane vibrations of laminated composite beams.

This paper focuses on using the IGA to analyze the dynamic behavior of laminated composite beams, specifically examining out-of-plane vibrations in beams with arbitrary layups.

2. Mathematical modeling

2.1. A brief introduction of B-spline and NURBS

2.1.1. B-spline

2.1.1.1. Knot vector

B-splines are defined on knot vectorΞ. The knot vector is a sequence of non-descending and consecutive real numbers in the parameter space, as follows:

$$\Xi = \{\xi_1, \xi_2, \dots, \xi_{n+p+1}\} \tag{1}$$

Here ξ_i is the i^{th} knot, p is the polynomial order and n is the number of basis functions.

2.1.1.2. B-spline basis functions

Given a knot vector, B-spline basis functions $N_{i,p}(\xi)$ of order p in the parameter space can be defined as follows:

$p = 0$:

$$N_{i,0}(\xi) = \begin{cases} 1 & if\ \ \xi_i \le \xi \le \xi_{i+1} \\ 0 & otherwise \end{cases}$$

$p \ge 1$:

$$N_{i,p}(\xi) = \frac{\xi - \xi_i}{\xi_{i+p} - \xi_i} N_{i,p-1}(\xi) + \frac{\xi_{i+p+1} - \xi}{\xi_{i+p+1} - \xi_{i+1}} N_{i+1,p-1}(\xi) \tag{2}$$

where i is the control point index.

2.1.2. B-spline curve

A piecewise B-spline polynomial curve is obtained by a linear combination of basic functions and control points, as presented below:

$$C(\xi) = \sum_{i=1}^{n} N_{i,p}(\xi) P_i \tag{3}$$

where $N_{i,p}(\xi)$ are B-spline basis functions defined on the knot vector, and $P_i \in R^d$ are the control point coordinates.

2.1.3. NURBS

B-splines are suitable for modeling complex shapes but do not have the ability to accurately represent conic sections. That is why extended B-splines, called NURBS, are used for precise drawing of simple shapes such as circles and ellipses. B-splines are piecewise polynomial functions, whereas NURBS are piecewise rational polynomial functions. All the features that were mentioned for B-splines also apply to NURBS.

2.1.3.1. NURBS functions

NURBS basis functions are defined as follows:

$$R_{i,p}(\xi) = \frac{N_{i,p}(\xi) w_i}{W(\xi)} \tag{4}$$

$$W(\xi) = \sum_{i=1}^{n} N_{i,p}(\xi) w_i \tag{5}$$

In the above equations, $N_{i,p}(\xi)$ are B-spline basis functions of degree p, and w_i are the weights of the NURBS.

2.1.4. NURBS curve

The NURBS curve, which is obtained by combining control points and NURBS basis functions, is expressed as follows:

$$C(\xi) = \sum_{i=1}^{n} R_{i,p}(\xi) P_i \tag{6}$$

Here $R_{i,p}(\xi)$ are NURBS basis functions and P_i are control points.

3. Isogeometric formulation

In IGA, to approximate the displacement field, NURBS basis functions are used. In the present problem, each control point has six degrees of freedom, including the axial displacementu_i, lateral and vertical deflections v_i andw_i, and three independent rotationsθ_i, φ_i, and ψ_i, where $i = 1,2,\dots,n$, in which n is the total number of control points. Therefore, the displacement fields of the beam elements are expressed as follows:

$$\begin{aligned} \theta^e &= \sum_{i=1}^{n} R_i \theta_i & u^e &= \sum_{i=1}^{n} R_i u_i \\ \varphi^e &= \sum_{i=1}^{n} R_i \varphi_i & v^e &= \sum_{i=1}^{n} R_i v_i \\ \psi^e &= \sum_{i=1}^{n} R_i \psi_i & w^e &= \sum_{i=1}^{n} R_i w_i \end{aligned} \tag{7}$$

In the above equation, R_i are the non-zero NURBS basis functions, and n is the number of non-zero basis functions on each element. Therefore, the degree of freedom vector for the beam element is expressed by Eq. (8):

$$\{\delta\} = \left\{ \begin{matrix} u_1, v_1, w_1, \varphi_1, \psi_1, \theta_1, u_2, v_2, w_2, \varphi_2, \psi_2, \\ \theta_2, \dots, u_n, v_n, w_n, \varphi_n, \psi_n, \theta_n \end{matrix} \right\}^T \tag{8}$$

Therefore, the strain potential and kinetic energy terms of each element can be expressed as follows:

$$U_B^{(e)} = \frac{1}{2}\{\delta\}^T[K_e]\{\delta\} \quad (9)$$

$$T_B^{(e)} = \frac{1}{2}\{\dot{\delta}\}^T[M_e]\{\dot{\delta}\} \quad (10)$$

in which $[K_e]$ and $[M_e]$ are the stiffness and mass matrices of the element. Finally, the equations governing the overall motion of the system in free vibrations are expressed as follows:

$$[M]\{\ddot{\Delta}\} + [K]\{\Delta\} = \{0\} \quad (11)$$

Assuming $\{\Delta\} = \{\Delta_0\}e^{\hat{\imath}\omega t}$ and $\lambda = \omega^2$, Eq. (11) is rewritten as follows:

$$([K] - \lambda[M])\{\Delta_0\} = \{0\} \quad (12)$$

where ω is the natural frequency of the desired beam and $\{\Delta_0\}$ is the corresponding mode shapes.

4. Results and discussion

Using the IGA, the vibration characteristics of a laminated composite beam with arbitrary layups are extracted. In all parts, the composite beam with a uniform rectangular cross-section with an aspect ratio of $L/h = 15$ and a unit width $(b = 1)$ is considered, also all layers are made of graphite/epoxy with the following mechanical properties:

$$E_{11} = 144.8\,GPa, \quad E_{22} = 9.65\,GPa, \quad G_{12} = G_{13} = 4.14\,GPa, \quad G_{23} = 3.45\,GPa, \quad \vartheta_{12} = 0.33, \quad \rho = 1389.23\ kg/m^3 \quad (13)$$

4.1. Convergence study

First, the convergence rate of the isogeometric solution using the h, p and k refinement methods is calculated and compared with the results of the reference [8]. The composite beams with asymmetric layering $[45/-45/45/-45]$ and two different boundary conditions are considered.

The open knot vector approach is used in all parts. The first five natural frequencies for the three refinement methods have been calculated. Increasing the number of elements, it is observed that in the IGA, the solutions converge faster compared to the finite element method.

4.2. Laminated composite beam with different layups

In this section, different layups and clamped–clamped boundary conditions have been studied. The first three natural frequencies for the in-plane, out-of-plane, torsional and axial vibrations have been presented. For example, in Table 1, the first three natural frequencies for out-of-plane bending vibrations are given. Three different layups namely cross ply, angle ply and unsymmetric have been considered. In the results obtained for symmetric lamina, it has been observed that by increasing the orientation of the fibers in angle-ply layups, the natural frequencies decrease.

Table 1. Three first dimensionless natural frequencies for out-of-plane modes

Mode No.	Cross ply		Angle ply $[\theta/-\theta]_s$		Unsymmetric	
	$[0]_4$	$[90]_4$	$\theta = 45$	$\theta = 60$	$[0/45]_2$	$[60/90]_2$
Ω_1	5.0285	1.6294	2.0512	1.6578	4.3726	1.6449
Ω_2	11.4228	4.3621	5.5444	4.4767	10.9643	4.4265
Ω_3	18.9674	8.2449	10.5950	8.5453	19.3870	8.4207

4.3. Effects of slenderness ratio, material anisotropy and effects of width to thickness ratio

The influence of slenderness ratio (L/h), material anisotropy (E_{11}/E_{22}) and width to thickness ratio (b/h) on the Dimensionless Fundamental Frequencies (DFF) of the beam for axial, in-plane bending, out-of-plane bending and torsional vibrations and with different boundary conditions and lamination scheme $[0^\circ\ 45^\circ\ 0^\circ\ 45^\circ]$ have been studied. For example, the influence of slenderness ratio on DFF of the beam for out-of-plane bending vibrations have been depicted in Fig. 1. It can be observed that the DFF in all directions decrease by increasing the slenderness ratio

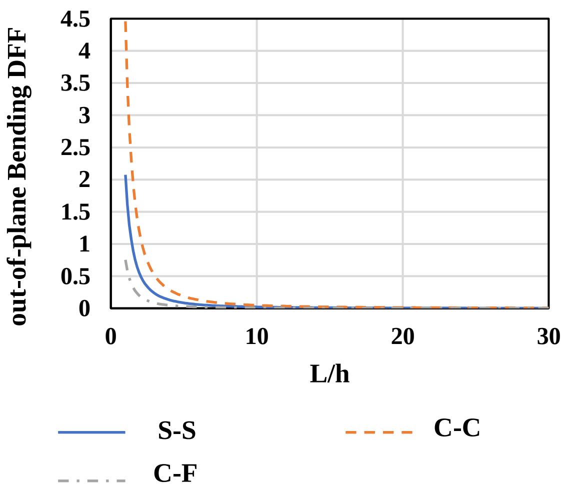

Figure 1. The influence of slenderness ratio (L/h) on the DFF of the laminated beam.

5. Conclusions

In order to find the free vibration response of the beam, the first order shear deformation theory has been used. The effect of transverse shear deformation, rotary inertia, as well as Poisson's effect and material couplings in axial, in-plane and out-of-plane displacements have been considered. In this way, all the vibration modes, i.e. in-plane, out-of-plane bending modes, axial and torsional modes of laminated composite beams with arbitrary layers have been obtained with the help of IGA.

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