

On the use of J-integral criterion for fracture assessment of cracked rigid polyurethane foam loaded in mixed mode I/II and I/III

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Received: 15/03/2024 Revised: 26/04/2024 Accepted: 26/05/2024

Abstract

The present study investigates and evaluates the J-integral criterion for rigid polyurethane foam using a cracked disk specimen subjected to bending in the mixed mode I/II and I/III loading conditions. The primary objective of this study is to assess a relationship for the J-integral based on a new approach considering the strain energy density in a linear elastic material loaded under mixed mode conditions. To do so, the relevant relationships for a sharp V-notch specimen are first derived and then extended to a cracked sample. The secondary aim of this research is to predict the fracture load of rigid polyurethane foams using the J-integral criterion in the mixed mode I/II and I/III loading conditions. To validate the criterion, finite element modeling of a cracked disk specimen is carried out and the fracture loads of the specimens under mixed mode I/II and I/III conditions are predicted according to the J-integral criterion. The results reveal the high accuracy of the criterion in predicting the fracture load of specimens under mixed mode conditions, with a maximum error percentage of 8%.

Keywords: Crack; J-integral; Rigid polyurethane foam; Mixed mode I/II and I/III; Average strain energy density; Finite element analysis

1. Introduction

Rigid polymer foams usually have a linear elastic behavior and are considered as a material with brittle behavior. Although this class of materials has a high strength in the compressive loading, the creation and growth of cracks in tensile loading in rigid foams is one of the major concerns in these materials [1]. Therefore, due to the wide applications of foam in engineering structures as well as high importance of safety of engineering structures, it is necessary to investigate the fracture of rigid foams with stress concentration factors such as cracks.

In the field of investigating the J-integral as a fracture criterion, Singh et al. [2] investigated the fracture behavior of a polycarbonate thermoplastic by performing fracture tests on single-edge grooved specimens and evaluated the J-integral criterion using the energy rate. Also, they showed that the value of J-integral is independent of the crack length and the length of sample, and the J-integral corresponds well with the value of critical stress intensity factor.

On the other hand, some researches have been carried out in the field of investigating the fracture of rigid polymer foams over the past years. Marsavina et

al. [3] succeeded in performing a fracture test of Asymmetric Semi-Circular specimens under Bending (ASCB)¹ made of rigid polyurethane (PUR) foam and under mixed in-plane loading (I/II) condition. In 2022, Imani et al. [4] subjected an Asymmetric Edge Notch Disk Bend (AENDB)² to asymmetric three-point bending. Also, the fracture toughness of mixed mode I/II and the fracture initiation angle was obtained for the above-mentioned fabricated samples. Recently, Vantadori et al. [5] in 2023 experimentally and numerically investigated the fracture toughness of a rigid polyurethane foam to study its dependence on the sample size. To conduct the experiment, they subjected rigid polyurethane foam with cracks of different sizes to the three-point bending loading and used the modified two-parameter model (MTPM)³ including the maximum applied load and the amount of crack opening (CMOD)⁴ to measure the fracture toughness.

In the current study, the J-integral criterion which is easy to use and has a relatively low computational cost is utilized. The J-integral criterion is used to predict the fracture of polyurethane foam with crack loaded in mixed modes I/II and I/III. For this purpose, first, J-integral relations under general mixed mode condition, including I/II and I/III, are obtained with a new

¹ Asymmetric Semi-Circular Bend

² Asymmetric Edge Notch Disk Bend

³ Modified Two-Parameter Model

⁴ Crack Mouth Opening Displacement

approach based on the strain energy density. Then, using the experimental results published in previous research, the accuracy of the relations is obtained and also, the efficiency of the J-integral criterion for predicting fracture in rigid polyurethane foams is evaluated.

2. Methodology

In this section, the J-integral relation is derived with a new approach based on the principles of strain energy density. It should be noted that in the sources of fracture mechanics, the J-integral relations for modes I, II and III of loading have been obtained using the concept of stress singularity around the crack tip [6].

In this research, first, the J-integral relation is used for the V-shaped notch, and then, is generalized for the crack (as a notch with a zero-degree notch angle). Finally, the J-integral value for a linear elastic material with a crack in plane strain condition for the mixed mode I/II and I/III is as follows:

$$J_{I/II} = \frac{1 - \nu^2}{E} [K_I^2 + K_{II}^2] \quad (1)$$

$$J_{I/III} = \frac{1}{E} [(1 - \nu^2)K_I^2 + (1 + \nu)K_{III}^2] \quad (2)$$

Where K_I , K_{II} and K_{III} in the above relations are the stress intensity coefficients of mode I, II, and III, respectively.

In order to check efficiency of using the J-integral as a fracture criterion in rigid foams, the previously experimental results of [7] were used, where they utilized a disk sample with an edge crack under bending (ENDB) made of rigid polyurethane foam with a density of 40 kg/m³ under the mixed modes I/II and I/III. The mechanical properties of the used foam are given in Table 1.

Table 1. Mechanical properties of rigid PUR foams [7]

σ_{ult} (MPa)	ν	E (MPa)
0.43	0.22	25

In the above table, E is the modulus of elasticity, ν is Poisson's coefficient and b is the final tensile strength of the foam. In figure (1), the geometry and how to load the geometry in the mixed modes I/II and I/III are shown schematically.

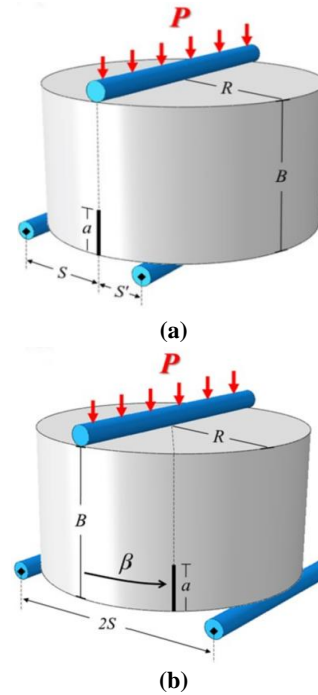


Figure 1. ENDB geometry used for (a) mixed mode I/II and (b) mixed mode I/III [7]

In order to evaluate the accuracy of the provided relations and the efficiency of the J-integral criterion in predicting fracture of cracked rigid polymer foams in the mixed mode, the finite element modeling of ENDB sample in the mixed modes I/II and I/III is performed by Abaqus software according to the experimental data presented in [7].

3. Results and Discussion

In order to verify the J-integral relations provided and predict the fracture loads in composite modes by the J-integral criterion, finite element simulations are performed and the J-integral values obtained from equations (1) and (2) are compared in Tables 2 and 3, in mixed modes I/II and I/III, respectively.

Table 2. Theoretical and numerical values of J-integral in the mixed mode I/II

S'/R	Numerical value (Pa.m)	Theoretical value (Pa.m)	Difference (%)
0.9	45.15	45.38	0.5
0.231	20.12	20.54	2.1
0.154	17.27	17.82	3.2
0.117	15.25	15.92	4.4
0.078	13.68	14.45	5.6

Table 3. Theoretical and numerical values of J-integral in the mixed mode I/III

β (deg)	Numerical value (Pa.m)	Theoretical value (Pa.m)	Difference (%)
0	45.15	45.38	0.5
30	18.28	18.63	1.9
45	14.88	15.42	3.6
55	13.03	13.67	4.9
63	11.54	12.28	6.4

The values presented in the last column in the above two tables show that the J-integral value from the energy approach has a very good agreement with the corresponding numerical values.

In addition, in Table 4 and Table 5, the critical loads (P_c) for the samples in mixed modes I/II and I/III, obtained numerically and theoretically, are presented.

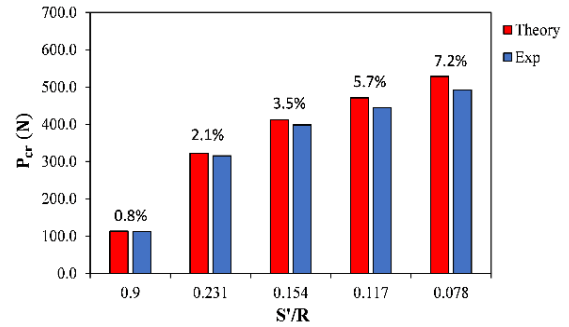
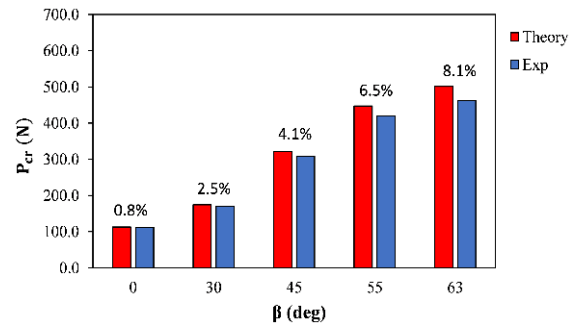
Table 4. Predicted fracture load of mixed mode I/II samples based on the J-integral criterion

P_c (N)	S'/R
112.9	0.9
321.8	0.231
412.0	0.154
470.5	0.117
528.0	0.078

Table 5. Predicted fracture loads of mixed mode I/III samples based on the J-integral criterion

P_c (N)	β (deg)
112.9	0
174.5	30
321.7	45
447.4	55
500.5	63

To check the accuracy of the results obtained using the J-integral criterion, the percentage difference in the fracture loads of the mixed modes according to the criterion compared to the corresponding experimental fracture load are shown in Figures (2) and (3), respectively.


Figure 2. Theoretical and experimental fracture load values and error percentage for the mixed mode I/II

Figure 3. Theoretical and experimental fracture load values and error percentage for the mixed mode I/III

4. Conclusions

In this research, first, J-integral relations for the cracked specimens loaded in mixed modes I/II and I/III were obtained with a new approach based on strain energy. Then, using the experimental data reported in a previous published article, a disk with an edge crack under bending in mixed modes I/II and I/III was simulated in Abaqus finite element software and the J-integral values were determined numerically and it was analyzed and verified numerically. The results showed the high accuracy of the relations presented for the J-integral value, so that the maximum difference with the numerical values was about 6%. Moreover, the effectiveness of the J-integral criterion for predicting the fracture load in rigid foam samples with cracks in the mixed modes I/II and I/III was investigated using previous experimental data. The results confirmed the accuracy of the J-integral criterion with the maximum percentage error of 8%. Therefore, the J-integral criterion is capable of predicting the fracture load of rigid polyurethane foam loaded under mixed modes I/II and I/III.

5. References

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