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Comparison of nonclassical controllers on piezoelectric nanoresonator: nonlinear frequency response and stability analysis

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Abstract

In current study, nonlinear vibrations and stability analysis of piezoelectric nanoresonator (PENR) considering with the effects of non-classical controllers such as strain gradient (SGT), nonlocal (NLT) and Gurtin–Murdoch surface/interface (GMSIT) theories are presented in comparison with the classical theory (CT). PENR subjected to nonlinear electrostatic excitation with direct (DC) and alternative (AC) voltages and also visco-pasternak medium. For this work, Hamilton’s principle and Galerkin technique are used to obtain the governing equations and boundary conditions and also to solve the equation of motion. Complex averaging method combined with arc-length continuation is used to investigate nonlinear frequency response and stability analysis of PENR. The results show that ignoring small-scale and surface/interface effects give inaccurate predictions of vibrational response of the PENR.

Keywords: Piezoelectric nanoresonator; Nonlocal strain gradient theory; Gurtin–Murdoch surface/interface; Nonlinear frequency response; Complex averaging method, Arc-length continuation.

1. Introduction

Nanotechnology is a branch of science that includes many fields of technology and science such as nanostructures, especially nano piezoelectric sensors/resonators, which are widely used in today's engineering and have significantly attracted the attention of researchers around the world due to their unique features and wide applications [1, 2]. On the other hand, due to the excessive use of nanosensor, especially piezoelectric nanosensor in vibration devices, mathematical modeling and analysis of their vibration behavior is necessary. For this purpose, non-classical theories have been presented to investigate nonlinear vibrations and dynamic analysis of nanostructures [3-5].

Based on the theory of non-local elasticity, Najafi et al. have investigated the effects of different parameters in the analysis of free vibrations of piezoelectric nanobeam [6]. Also, in the study conducted by Arefi, it has been shown that the increase of the non-local parameter leads to the increase of rotations, in-plane displacements and transverse deflection of a nano-piezoelectric shell with double curves [7]. Ebrahimi et al. have used the nonlocal strain gradient theory to investigate the vibration analysis of viscoelastic nanobeams [8]. Using the nonlocal strain gradient theory and the analytical method of multiple time scales, the nonlinear vibrations of the nonlocal Euler-Bernoulli nanowire as a nanoelectromechanical structure have been investigated by Karamad et al. [9]. Also, according to the Gurtin–Murdoch surface/interface theory, nonlinear buckling-postbuckling of piezoelectric nanostructures have been investigated by Fang et al [10]. Recently, Hashemi Kachapi et al. some important analytical methods on a small scale, such as Gurtin–Murdoch surface/interface energy theories, Eringen's non-local theory and non-local strain gradient, as well as the combination of these different methods. have presented to investigate the effects of non-classical controllers on natural frequencies, nonlinear vibrations and stability analysis of multi-walled piezoelectric nanostructures under various excitations such as harmonic, visco-pasternak and nonlinear electrostatics [11-16].

It should be noted that a very limited number of studies simultaneously studied the effect of surface/interface energy and small-scale effects for nanostructures, especially piezoelectric nanostructures. By using non-local effects and surface energy, buckling analysis of piezoelectric nanoshells under external voltages and compressive loads has been investigated by Sun et al [17]. Also, Kiani has studied the post-buckling analytical response of beam-like nanostructures by considering surface and non-local effects [18].

In all the previous works that have been done by the authors, very few researches have been done in the analysis of vibrations and stability of piezoelectric nanostructures by simultaneously considering the effects of strain gradient, Gurtin–Murdoch surface/interface and non-local effects. The current research is a continuation of the work done [13], but the subject of the study, especially the applied forces, is completely different from the previous work, and as a result, different results are obtained from the previous article, and unlike the previous work, the present nanostructure is simultaneously subjected to nonlinear electrostatic stimulation with direct (DC) and alternating (AC) voltages as well as Visco Pasternak medium.

2. Mathematical formulation and Solution procedure

A piezoelectric nanoresonator based on cylindrical nanoshell subjected to visco-pasternak medium and nonlinear electrostatic excitation is shown in Figure 1. All of the physical and geometrical properties of the mentioned nanostructures can be seen in reference Hashemi Kachapi et al. [12].

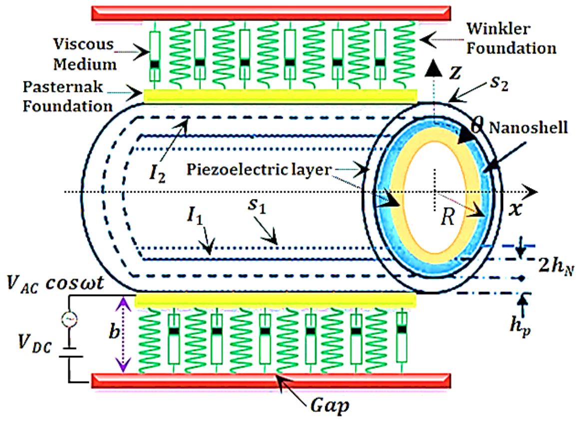

Figure 1. A piezoelectric nanoresonator subjected to nonlinear electrostatic excitation

The governing equations of motion and corresponding boundary conditions of the piezoelectric shell are obtained by applying the following Hamilton principle:

$$\int_0^t (\delta T - \delta\pi + \delta w_{vf} + \delta w_e)dt = 0, \qquad (1)$$

For this purpose, the total strain energy of PENS considering the surface/interface effect can be presented as:

$$\pi = \frac{1}{2}\int_0^L \int_0^{2\pi} \left\{ \begin{matrix} N_{xx}\varepsilon_{xx}^0 + N_{\theta\theta}\varepsilon_{\theta\theta}^0 \\ +N_{x\theta}\gamma_{x\theta}^0 + M_{xx}\kappa_{xx} \\ +M_{\theta\theta}\kappa_{\theta\theta} + M_{x\theta}\kappa_{x\theta} \\ +\eta_{33}\bar{E}_{zp}^2 h_p \end{matrix} \right\} Rd\theta dx \qquad (2)$$

In Eq. (2), the forces (N) and moment (M) resultants are determined in [12-14]. The first variation of kinetic energy of the PENS can be written as:

$$\delta\int_{t_1}^{t_2} Tdt = -\int_{t_1}^{t_2}\iint \left\{ I\left(\left(\frac{\partial^2 u}{\partial t^2}\right)\delta u + \left(\frac{\partial^2 v}{\partial t^2}\right)\delta v + \left(\frac{\partial^2 w}{\partial t^2}\right)\delta w \right) \right\} Rd\theta dxdt \qquad (3)$$

where

$$I = \int_{-h_N}^{h_N} \rho_N\, dz + \int_{-h_N-h_p}^{-h_N} \rho_p\, dz + \int_{h_N}^{h_N+h_p} \rho_p\, dz + \rho^{S,I} = 2\rho_N h_N + 2\rho_p h_p + 2\rho^S + 2\rho^I \qquad (4)$$

Also, and first variation of the work done by the viscoelastic foundation and nonlinear electrostatic excitation, respectively, can be written as [12-14]:

$$\delta W_{vf} = -\int_0^L \int_0^{2\pi} \int_0^w \left(\begin{matrix} K_w w \\ -K_p \nabla^2 w \\ +C_w \dfrac{\partial w}{\partial t} \end{matrix} \right) \delta w\, Rd\theta dx, \qquad (5)$$

$$\delta W_e = \int_0^L \int_0^{2\pi} \int_0^w \frac{\pi\Upsilon (V_{DC} + V_{AC}cos(\omega t))^2}{\left(\begin{matrix} \sqrt{\begin{matrix}(b-w) \\ \times (2R+b-w)\end{matrix}} \\ \times \left[\cosh^{-1}\left(\dfrac{1+}{\dfrac{b-w}{R}}\right)\right]^2 \end{matrix} \right)} \delta w\, Rd\theta dx \qquad (6)$$

It is important to note that all relationships, coefficients and phrases for nonlocal strain gradient, surface/interface theories and nonlocal strain gradient surface/interface and small-scale stress-strain relationships and etc. can be seen in full detail in [12-14].

By applying the Galerkin method, Hamilton's principle, least-squares polynomial format of electrostatic force, using the displacement and shear deformation in the assumed mode method, applying non-dimensional strain and kinetic energies and also non-dimensional works and then substituting in the Lagrange-Euler equations, the dimensionless governing equations of motion and boundary conditions for PENS are obtained to the following equations:

$$\begin{aligned} &[(K)_u^u + (K_{bc})_u^u]\{\bar{u}\} + [(K)_u^v + (K_{bc})_u^v]\{\bar{v}\} \\ &\quad + [(K)_u^w + (K_{bc})_u^w]\{\bar{w}\} \\ &\quad + [(NL)_u^w + (NL_{bc})_u^w]\{\bar{w}^2\} \\ &\quad = [(M)_u^u]\{\ddot{\bar{u}}\} + \bar{F}_{up}^{bc}, \end{aligned} \qquad (7)$$

$$
\begin{aligned}
&[(K)_v^u + (K_{bc})_v^u]\{\bar{u}\} + [(K)_v^v + (K_{bc})_v^v]\{v\} \\
&\quad + [(K)_v^w + (K_{bc})_v^w]\{\bar{w}\} \\
&\quad + [(NL)_v^w + (NL_{bc})_v^w]\{\bar{w}^2\} \\
&\quad = [(M)_v^v]\{\ddot{\bar{v}}\} + \bar{F}_{vp}^{bc},
\end{aligned} \tag{8}
$$

$$
\begin{aligned}
&[(K)_w^u]\{\bar{u}\} + [(K)_w^v]\{\bar{v}\} \\
&+ \left[(K)_w^w + (K_{bc})_w^w - \left(K_{vp}\right)_w^w - (K_{e2})_w^w\right]\{\bar{w}\} \\
&+ [(NL)_w^u + (NL_{bc})_w^u]\{\bar{w}\bar{u}\} \\
&+ [(NL)_w^v + (NL_{bc})_w^v]\{\bar{w}\bar{v}\} \\
&+ [(NL)_{w2}^w + (NL_{bc})_{w2}^w - (NL_{2e})_{w2}^w]\{\bar{w}^2\} \\
&+ [(NL)_{w3}^w + (NL_{bc})_{w3}^w - (NL_{3e})_{w3}^w]\{\bar{w}^3\} \\
&= ([(M)_w^w + (M_{bc})_w^w])\{\ddot{\bar{w}}\} + ([(C)_w^w] \\
&+ [(C_{bc})_w^w])\{\dot{\bar{w}}\}) + \bar{F}_{wp} + \bar{F}_{wp}^{bc} - \bar{F}_{we} \\
&- \bar{F}_e \left\{ \begin{matrix} ((\bar{V}_{AC}\cos\bar{\varpi}\tau)^2 + 2\bar{V}_{AC}\bar{V}_{DC}\cos\bar{\varpi}\tau) \times \\ \left(\begin{matrix} \bar{C}_4(NL_e)_{w3}^w + \bar{C}_3(NL_e)_{w2}^w \\ +\bar{C}_2(K_e)_w^w + \bar{C}_1\bar{F}_1 \end{matrix} \right) \end{matrix} \right\}
\end{aligned} \tag{9}
$$

All steps to obtain the governing equations and also all coefficients and phrases in equations (7)- (9) are defined in [12-14]. In order to study the nonlinear dynamics response, arc-length continuation and complex averaging approaches are presented [12-14, 19].

Verification, comparison and convergence study is investigated with full details for PENS in [12-14]. In this section, pull-in instability analysis on dimensionless natural frequency and nonlinear dynamics response are presented. For this purpose, different boundary condition such as clamped edge (CC), simply supported edge (SS), clamped-simply supported edge (CS) and, clamped-free edge (CF) is presented.

3. Results and discussions

The surface and bulk material properties of Aluminum (Al) nanoshell and PZT piezoelectric layer, others physical and geometrical parameters of PENR and also two case of surface density are shown in Table 1 [12-14].

Table 1. Surface and bulk properties of Al and PZT-4 and other material and geometrical parameters

E_N (GPa)	υ_N	ρ_N $(\frac{kg}{m^3})$	λ^I (N/m)	μ^I (N/m)	τ_0^I (N/m)	ρ^I (kg/m^2)
70	0.33	2700	3.786	1.95	0.9108	5.46×10^{-7}

C_{11p} (GPa)	C_{22p} (GPa)	C_{12p} (GPa)	C_{21p} (GPa)	C_{66p} (GPa)	E_p (GPa)
139	139	77.8	77.8	30.5	95
υ_p	ρ_p $(kg\,m^{-3})$	η_{33p} $(10^{-8}$ $F/m)$	$\lambda^S(N/m)$	$\mu^S(N/m)$	τ_0^S (N/m)
0.3	7500	8.91	4.488	2.774	0.6048
e_{31p} (C/m^2)	e_{32p} (C/m^2)	e_{31p}^S (C/m)	e_{32p}^S (C/m)	$\rho^S(kg/m^2)$	
-5.2	-5.2	-3×10^{-8}	-3×10^{-8}	5.61×10^{-6}	
$R(m)$	L/R	h_N/R	h_p/l	b/R	$C_w\left(\frac{N.S}{m}\right)$
1×10^{-9}	10	0.01	0.005	0.1	1×10^{-3}
$K_w(N/m^3)$	$K_p(N/m)$	$V_p(V)$	V_0	$V_{DC}(V)$	$V_{AC}(V)$
9×10^{17}	2.07	1×10^{-5}	1	1.5	0.5
$\mu(m^2)$	$\eta(m^2)$				
$(1 \times 10^{-10})^2$	$(1 \times 10^{-11})^2$				

Case 1		Case 2	
$\rho^I(kg/m^2)$	$\rho^S(kg/m^2)$	$\rho^I(kg/m^2)$	$\rho^S(kg/m^2)$
5.46×10^{-7}	5.61×10^{-6}	5.46×10^{-8}	5.61×10^{-7}

The main purpose of this section is to compare three nonclassical theories of NLT, SGT and GMSIT with classical theory CT. For this purpose, the effect of different geometrical parameters and material with and without strain gradient, nonlocal and surface/ interface effects will be analyzed on dimensionless natural frequency (DNF), stability analysis and nonlinear frequency response using arc-length continuation as numerical method for the PENR with specifications mentioned to Table 1.

The comparison of three nonclassical theories of NLT, SGT and GMSIT with classical theory CT on nonlinear frequency response of SS and CC PENR with $\bar{V}_{DC} = 1.7$ and $\bar{V}_{AC} = 0.5$ respectively are presented in Figures 2 and 3. It can be seen from Figure 2 that considering the frequency analysis results and the rigidity of SS PENR with regard to the type of theory, the consideration of the S/I effects in the GMSIT cas2 (case1) theory leads to hardening (softening) the PENR, and by increasing (decreasing) the resonant frequency, it reduces the resonance amplitude of PENR. Also, instability (due to the amounts of voltages applied to it) does not occur in the system. In other theories (CT, NLT, and SGT), instability with saddle-node bifurcations and nonlinear hardening behavior occurs and it is clear that in the non-classic theories of NLT and SGT, the oscillation amplitude, the range of instability is greater than that of the classical one, and with increasing $\bar{\tau}$ and $\bar{\eta}$, the amplitude of the oscillation and the range of instability increases, but the resonance frequency decreases.

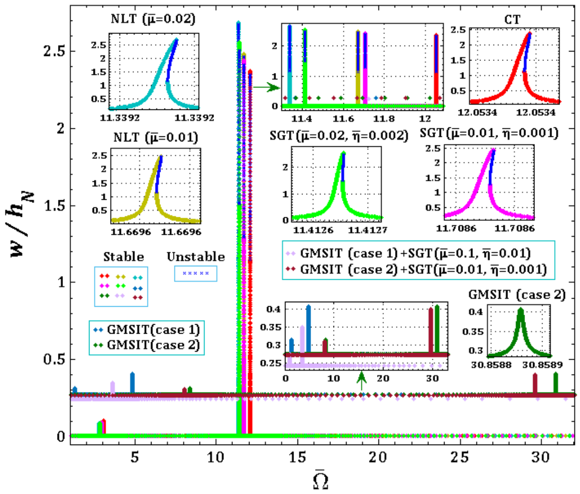

Figure 2 Comparison of nonclassical theories with classical theory on the nonlinear vibration and stability analysis of SS PENR

Furthermore, from Figure 3, it can be seen that with increasing of dimensionless nonlocal parameter, the resonance amplitude decreases. These observations mean that the small scale effects in the nonlocal model make CC PENR more flexible. In most cases, CC boundary condition have results similar to the SS boundary condition. Only in the SGT theory, with increasing of $\bar{\tau}$ and $\bar{\eta}$, the system in CC boundary condition exhibits a nonlinear softening instability behavior, while in the SS boundary condition, PENR has a nonlinear hardening instability behavior.

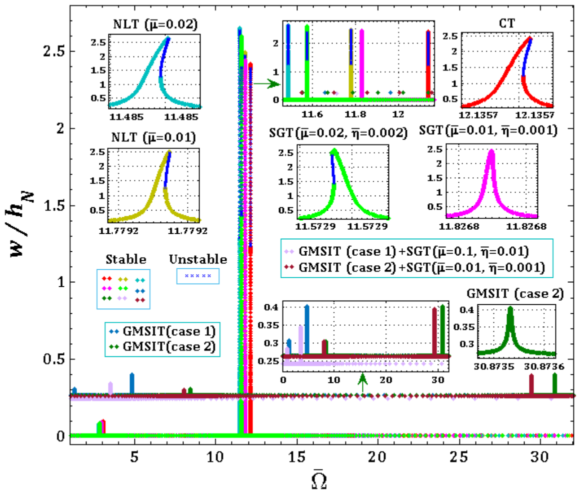

Figure 3. Comparison of nonclassical theories with classical theory on the nonlinear vibration and stability analysis of CC PENR

4. Conclusions

In current study, nonlocal, strain gradient and Gurtin–Murdoch surface/interface theories are presented to investigate nonlinear vibration and stability analysis of piezoelectric nanoresonator subjected to nonlinear electrostatic excitation compared to classical theory. For this analysis, Hamilton's principle, Galerkin technique and also Complex averaging method combined with arc-length continuation are used to compare nonclassical theories with classical one for nonlinear frequency response and stability analysis of the PENR.

The results indicated that the natural frequency of the classical theory is greater than the NLT and SGT theories, indicating a reduction in the rigidity of the system due to the consideration of these two theories. The NLT theory has a higher frequency than SGT theory due to its increased rigidity. Taking into account surface / interface effects and using surface / interface densities higher or lower can create more or less frequencies than classical ones. The consideration of the S/I effects in the GMSIT case2 (case1) theory leads to hardening (softening) the PENR, and by increasing (decreasing) the resonant frequency, it reduces the resonance amplitude of SS PENR and instability does not occur in this case. Also, in CT, NLT, and SGT, instability with saddle-node bifurcations and nonlinear hardening behavior occurs and it is clear that in the non-classic theories of NLT and SGT, the oscillation amplitude, the range of instability is greater than that of the classical one.

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