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Dynamics of Confined Spherical Gas Bubbles in an Elastic Vessel Filled with a Thixotropic Fluid

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Abstract

In this paper, the dynamics of a tiny spherical gas bubble surrounded by an incompressible liquid confined in a linearly-elastic vessel has been numerically investigated. To unbalance the bubble, the vessel is deformed which increases the pressure of the liquid and thereby forcing the bubble to radially deform. The liquid is assumed to be thixotropic obeying the Moore model as a good representation of both thixotropic and shear thinning behavior simultaneously. For small Thixotropy numbers, thixotropic effects dominate whereas for large Thixotropy numbers, shear-thinning dominates. After deriving the integro-differential equations governing the bubble dynamics in Moore liquid, they have been numerically solved using ODE23s solver in the MATLAB software. Based on the obtained numerical results, an increase in the Thixotropy number leads to a sharp growth in the bubble's oscillations. By increasing the viscosity ratio of the Moore model or the elasticity of the vessel the amplitude of the bubble oscillations is diminished. To minimize the radial stress exerted on the vessel by the oscillating liquid, Thixotropy number should be as large as possible. In cases where this is not feasible, the surface tension of the liquid should be reduced using appropriate surfactants.

Keywords: Gas Bubble Dynamics, Thixotropic Fluid, Thixotropy Number, Elasticity, Surface Tension, Radial Stress.

1. Introduction

Bubble dynamics has always been a topic of prime interest due mainly to its application in cavitating flows [1]. In recent years, however, interest in bubble dynamics has significantly increased when it has been discovered that tiny gas bubbles can be used as contrast agent in sonography and also for drug dlivery[2]. Due to its industrial and biomechanical applications, a variety of different scenarios (involving free and encapsulated bubbles) have been investigated in the past [3-5]. In the majority of cases, the bubble is not confined. In the present study, we are primarily interested in a situation in which a bubble is formed (e.g., by laser beam) in the middle of an elastic cavity and has reached an equilibrium radius controlled by the interfacial tension. In recent years, it has been shown that such a bubble can become unstable if the elastic vessel is slightly deformed [6, 7].

In the above studies, the liquid surrounding the bubble was Newtonian. The question then arises as to what will happen if the liquid is non-Newtonian. In a recent work, Arefmanesh et al. [8] extended the analysis to shear-thinning fluids obeying the power-law model and reached to the conclusion that the effect of shear-thinning is noticeable. In the present work, we further extend their work to thixotropic fluids. For ease of analysis we rely on the Moore rheological model to represent thixotropic liquids. Our approach is numerical. Our objective is to see what happens to the bubble when it becomes unstable due to the small deformation of the vessel. The effect of Moore model parameter are then investigated on the amplitude of the free oscillation generated this way. We are also primarily interested in figuring out ways by which the radial stress experienced by the vessel inner boundary can be minimized.

2. Mathematical Formulation

Figure 1 shows the flow geometry. The geometry comprises an elastic cavity that is initially filled with a viscous liquid. This figure shows a situation in which, at $t > 0$ a spherical gas/vapor bubble has been formed in the center of the cavity with radius, $R_b(t)$ which is determined by the gas/vapor pressure and the surface tension.

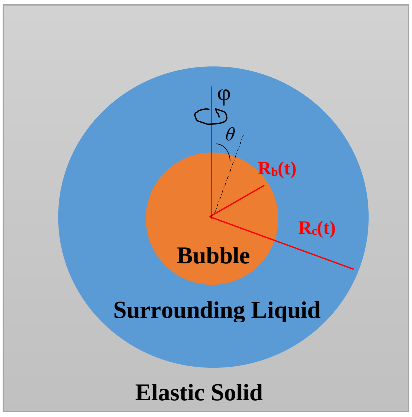

Figure 1. Schematic of configuration of a gaseous bubble (radius $R_b(t)$) at t > 0 surrounded by finite layer of a thixotropic liquid coated by a linear elastic solid (radius $R_c(t)$).

Assuming that the liquid surrounding the bubble is incompressible, the bubble can be forced to expand or shrink by deforming the elastic vessel. By so-doing, the pressure transmitted to the liquid at its interface with the solid is simultaneously sensed by the bubble and it responds by changing its radius, $R_b(t)$. A radial flow is also induced in the liquid in such a way that its volume is conserved at all times. Assuming that the solid vessel is a linearly elastic material, any change in its volume at t = 0 can unbalance the bubble because it increases the pressure of the liquid surrounding the bubble. The following expression is used for the rise in the liquid pressure when vessel volume is varied [7]:

$$\forall_c(t) - \forall_{c,0} = \frac{\forall_{c,0}}{K_c}\left(p_{l,c} - p_{l,0}\right) \tag{1}$$

Where K_c stands for the rigidity of the vessel. In this relationship subscript l referring to the liquid. Since the liquid is incompressible, the change in bubble volume is the same as the change in vessel volume. As such, the pressure rise in the liquid can be related to the bubble volume as a function of time [7]:

$$p_{l,c} - p_{l,0} = \frac{\forall_b(t) - \forall_{b,0}}{\forall_{c,0}} K_c \tag{2}$$

With subscript B denoting bubble respectively.

Considering purely radial motion of the bubble, for the liquid, the radial velocity should satisfy the mass conservation equation in spherical coordinate. The radial velocity in the liquid can be related to the bubble radius as:

$$v_r(r,t) = \frac{dR}{dt}\left(\frac{R}{r}\right)^2 \tag{3}$$

Where bubble radius, R_b, has been replaced by R for simplicity. The liquid is assumed to obey the Moore thixotropic model as follows [9]:

$$\eta = \eta_\infty(1 + \alpha S) \tag{4}$$

where η, α and S denote fluid's apparent viscosity, viscosity-gap ratio, and structural parameter, respectively. The subscript ∞ refers to the case of S = 0 where all microstructures are broken. For the case of S = 1 (i.e., when all microstructures are rebuilt) we use subscript zero for referring to the zero-shear viscosity as $\eta_0 = \eta_\infty(1+\alpha)$. Evidently, $\alpha > 1$ denotes the difference between these two limiting viscosities in the Moore model. The structural parameter is expressed as [9]:

$$\frac{DS}{Dt} = a(1 - S) - bS\dot{\gamma} \tag{5}$$

With S(0) = 1 as initial condition due to the first order nature of Equation 5 respect to time (as a good representation of fully complete structures before starting the motion). $D/Dt = \partial/\partial t + V.\nabla$ shows material derivative. a and b are controlling parameters of the model denoting the rate of fully breakdown and fully rebuild of polymer chains respectively. $\dot{\gamma}$ also represents fluid's shear rate. Direct substitution of Eq. 3 into radial momentum equation then integrating on the confined liquid subspace from bubble surface to the coating surface leads to following second order ODE governing the bubble's radial oscillations:

$$\begin{aligned}
&\rho\left[R\frac{d^2R}{dt^2} + 2\left(\frac{dR}{dt}\right)^2\right]\left(1 - \frac{R}{R_c}\right) + \\
&\frac{\rho}{2}\left(\frac{dR}{dt}\right)^2\left(\frac{R^4}{R_c^4} - 1\right) = \\
&-4\eta_\infty\frac{dR}{dt}\frac{R^2}{R_c^3}(1 + \alpha S)\bigg|_{R_c} - p_{l,0} - \\
&K_c\frac{R^3 - R_{b,0}^3}{R_{c,0}^3} - 2\frac{\sigma}{R} + p_v + \\
&p_{0,g}\left(\frac{R_{b,0}}{R}\right)^{3\gamma} - 12\eta_\infty\frac{dR}{dt}R^2\int_R^{R_c}(1 + \alpha S)\frac{dr}{r^4}
\end{aligned} \tag{6}$$

As the liquid confined between the bubble and vessel is incompressible, time variation of vessel radius as a direct conclusion from radial deformation of the system could be expressed as:

$$R_c(t) = \left[R_{c,0}^3 + (R(t)^3 - R_0^3)\right]^{\frac{1}{3}} \quad (7)$$

That is worth to mention that equation reduces to the classic Rayleigh-Plesset equation [3] by simply setting α = 0. In this equation, σ, p_v and $p_{0,g}$ denote surface tension, vapor pressure and initial gas pressure inside the bubble. The initial conditions are: $R(0) = R_0, \dot{R}(0) = 0$. To benefit from working with dimensionless numbers, we define:

$$\mathrm{Re} = \frac{\sqrt{\rho_0 p_{l,0}} R_{b,0}}{\eta_\infty}, We = \frac{2\sigma}{R_{b,0} p_{l,0}},$$
$$\xi = \frac{K_c}{p_{l,0}}, Tx = b, \quad (8)$$
$$\Psi = \frac{1}{a R_{b,0}} \sqrt{\frac{p_{l,0}}{\rho_0}}, \zeta = \frac{p_v}{p_{l,0}}, C_f = \frac{p_{0,g}}{p_{l,0}}.$$

where Re, We, ξ, Tx, Ψ, ζ and C_f represent Reynolds number, Webber number, Elasticity number, Thixotropy number (also called Deborah number [9]), destruction number, vapor pressure number, and gas pressure number, respectively.

3. Numerical Method

The governing equations (Eqs. 5 and 6) subjected to the corresponding initial conditions are solved numerically using MATLAB ODE 23s solver after a suitable change of variable which transfers time-dependent radial coordinate to a fixed coordinate as:

$$Z(r,t) = \frac{R_{c,0}}{R_{b,0}} \left(\frac{r - R}{R_c - r}\right) \quad (9)$$
$$\mathrm{R(t)} \le \mathrm{r} \le \mathrm{R}_c(t) \rightarrow 0 \le \mathrm{Z} \le \infty$$

The governing equations obtained based on this idea have been discretized using the finite difference (second order) scheme. That is worth to say that COMSOL Multiphysics has also been used for further comparison purpose and so good agreement was seen between both set of the solvers.

4. Results and Discussion

In this section we present a summary of the numerical results. As earlier mentioned, we are primarily interested on the parameters of the Moore model on the bubble dynamics. But, for completeness, we also investigate the effect of several other key parameters such as vessel elasticity. Figure 2 shows the effect of the Thixotropy number (Tx) on the bubble response, which is predicted to be oscillatory. As is seen in this figure, by increasing the Tx number, the amplitude of the free oscillations is increased. Since smaller Tx numbers refer to strongly-thixotropic fluids, one can conclude that thixotropic behavior of physiological fluid has a negative effect on bubble dynamics because a larger bubble can damage the inner wall of the vessel (to be shown shortly). This is not surprising because a large Tx means a smaller apparent viscosity so that momentum transfer between liquid's layers is facilitated.

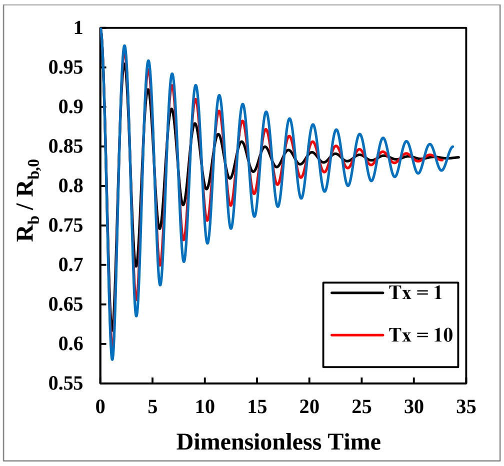

Figure 2. Effect of the Thixotropy number on time response of the bubble's oscillations obtained at: α = 2, ξ = 1000, ψ = 1, φ =1/7, Re = 50, We = 0.2, ζ =0.023, C_f = 0.001

Figure 3 shows the effect of the viscosity ratio of the Moore model (α) on the bubble response. As is seen in this figure, this ratio (which is zero for Newtonian fluids) has a retarding effect on the bubble, simply because it gives rise to a larger apparent viscosity.

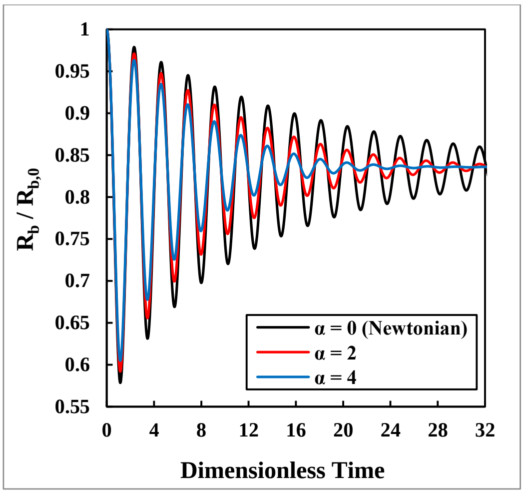

Figure 3. Effect of viscosity ratio of Moore model on time response of the bubble's oscillations obtained at: Tx = 10, ξ = 1000, ψ = 1, φ =1/7, Re = 50, We = 0.2, ζ =0.023, C_f = 0.001

Finally in Fig. 4 we have investigated the effect of the Thixotropy number on the maximum radial stress applied from the liquid side on the inner layer of the elastic vessel for different values of the gas/liquid surface tension. According to this figure, for any given Weber number, by increasing the Tx number the maximum stress is slightly increased until it reaches a peak beyond which it sharply decreases. This suggests that, in order to avoid the maximum stress applied on the vessel, the thixotropy number should be either smaller or larger than the critical Tx, with the latter being more appropriate although it means weakly thixotropic (but strongly shear-thinning type of the Moore model). Since in real world we do not have much control on the severity of thixotropic behavior in physiological fluids such as synovial fluid, we need a passive means to reduce the maximum stress. Based on Fig. 4, we can use appropriate surfactants for this purpose. That is to say that, by reducing the surface tension, the maximum stress is dramatically decreased.

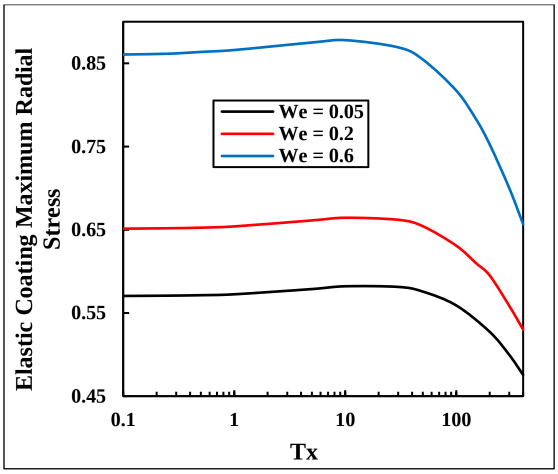

Figure 4. Effect of Weber number on variations of elastic coating maximum radial stress with Thixotropy number obtained at: α = 2, ξ = 1000, ψ = 1, φ =1/7, Re = 50, ζ =0.023, C_f = 0.001

5. Conclusion

In present study, we have numerically studied the effect of a fluid's thixotropic behavior on the dynamics of a spherical gas bubble confined in a closed-ended flexible cavity made of a linearly-elastic material. The bubble is forced to become unstable through a sudden change in the volume of the vessel. This perturbation sharply increases the liquid's pressure at liquid/solid interface which is then transmitted to the bubble and forces it to undergo free oscillations. Based on the obtained numerical results, the following conclusions can be made:

- Thixotropic number amplifies the amplitude of bubble's free oscillations.
- An increase in the viscosity ratio of the Moore model has a retarding effect on the bubble's oscillations.
- To minimize the radial stress experienced by the vessel (which, in physiological systems, may give rise to tissue damage and/or bleeding) it is predicted that the Thixotropy number should be as large as possible, which tacitly means that the fluid should not be too thixotropic. In cases there is no control on the degree of a fluid's thixotropy, it is predicted that we should decrease the surface tension at liquid/bubble interface using appropriate surfactants.

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6. References

[1] Dollet B, Marmottant P, Garbin V (2019) Bubble dynamics in soft and biological matter. *Annual Review of Fluid Mechanics* 51: 331-355.

[2] Nesser HJ, Karia DH, Tkalec W, Pandian NG (2002) Therapeutic ultrasound in cardiology. *Herz* 27(3): 269-278

[3] Plesset MS, Prosperetti A (1977) Bubble dynamics and cavitation. *Annual Review of Fluid Mechanics* 9: 145–185.

[4] Martynov S, Stride E, Saffari N (2009) The natural frequencies of microbubble oscillation in elastic vessels. *The Journal of the Acoustical Society of America* 126(6): 2963–2972.

[5] Duncan JH, Milligan CD, Zhang SG (1996) On the interaction between a bubble and a submerged compliant structure. *Journal of Sound and Vibration* 197(1): 17–44.

[6] Vincent O, Marmottant P, Gonzalez-Avila SR, Ando RK, Ohl CD (2014a) The fast dynamics of cavitation bubbles within water confined in elastic solids. *Journal of Soft Matter* 10: 1455–1461.

[7] Wang QX (2017) Oscillation of a bubble in a liquid confined in an elastic solid. *Physics of Fluids* 29 (7): 072101.

[8] Arefmanesh A, Madandar Arani M, Abbasian Arani A (2022) Dynamics of a bubble in a power-law fluid confined within an elastic Solid. *European Journal of Mechanics B Fluids* 94: 29–36.

[9] Derksen JJ (2011) Simulations of Thixotropic Liquids. *Applied Mathematical Modelling* 35(4):1656-1665.