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Nonlinear dynamic response of truncated conical shells reinforced with carbon nanotubes with functional graded ceramic-metal matrix under harmonic excitation

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Abstract

This paper analyzes the nonlinear dynamic response of truncated conical shells reinforced with carbon nanotubes with functional graded ceramic-metal matrix subjected to harmonic excitation. Carbon nanotubes are distributed with three different patterns along the length and thickness of the conical shell. The matrix material of the shell is considered to be a combination of metal and ceramic, whose properties change as a power function along the thickness of the shell. In order to analyze the dynamic of this system, firstly, the nonlinear dynamic equations of the conical shell are derived based on the first order shear deformation theory and von Karman's strain-displacement relations. Then, with the help of Galerkin discretization method, partial differential equations of the system are converted into time-dependent ordinary differential equations. Adams-Bashforth numerical method is used to solve the system of nonlinear differential equations. Finally, a parametric study is presented to investigate the effects of some parameters of the system, such as the power index, volume fraction and distribution pattern of carbon nanotubes, the geometric characteristics of the shell, and amplitude of the excitation force on the nonlinear dynamic response of the conical shell. In order to validate, the results of this article are compared and presented with the results of previous valid references.

Keywords: Nonlinear dynamic response, Truncated conical shell, Functionally graded carbon nanotubes, Adams-Bashforth method.

1. Introduction

Conical shells as a common structure composed of various materials are widely used in many structures such as pressure vessels, spacecraft, submarines and other engineering structures. Analysis of buckling, vibrations and dynamic response of conical shells is of particular importance. Therefore, many studies have been done in this field. Bochkarev et al. [1] based on the classical theory, investigated the vibrations of incomplete conical shells containing fluid using the generalized differential quadratic method. Bagheri et al [2] investigated the free vibrations of conical-spherical composite shells based on the theory of first-order shear deformation and the generalized differential quadratic method. Amabili and Balasubramanian [3] investigated the nonlinear forced vibrations of multilayer composite conical shells using modified shear deformation theory.

Functionally graded materials (FGMs) are very important in industry due to their unique characteristics and properties. These materials, which have continuous and gradual changes in properties in their structure, are used in many industries. Therefore, due to the special importance of these materials, many studies have been conducted on the dynamic and vibrational behavior of structures of this type. For example, Shadmani et al. [4] investigated the nonlinear free vibrations of conical shells made of functionally graded materials with variable properties in two directions using the improved Lindstedt Poincaré method. In another research, Shadmani et al. [5] investigated the effect of thermal environment on the nonlinear natural frequency of bidirectional functional scaled conical shells. Youseftabar et al. [6] investigated the effect of porosity with uniform and non-uniform distribution on the nonlinear natural frequency of bidirectional functional scaled conical shells using the harmonic balance method and based on the theory of first-order shear deformation.

In 1991, carbon nanotubes (CNTs) were discovered, and soon after, researchers and industrialists realized the remarkable electrical, thermal, and mechanical properties of these materials. CNTs have been considered as an effective reinforcement due to their high strength, high stiffness, low weight and large aspect ratio. Therefore, a lot of research has been done on the vibrational and dynamic behavior of structures reinforced with carbon nanotubes. Chakraborty et al. [7] investigated the effect of CNTs on the stability and vibration characteristics of plates and panels placed in a thermal environment. Khalaf and Hasan [8] analyzed the nonlinear forced vibrations of FG hybrid three-phase nanocomposite annular shell fragments reinforced by CNTs and graphene nanoplatelets.

Existing research shows that adding CNTs to metals, ceramics and polymers can significantly improve their properties. Based on these advantages, DFG-CNT materials composed of CNTs and FGMs with ceramic-metal base can greatly improve the mechanical properties of the shell structure of FGMs. By reviewing the previous literature, it was found that the nonlinear dynamic response of truncated conical shells reinforced with CNTs with a FG ceramic-metal matrix under harmonic excitation has not been reported. Therefore, the present study introduces a new investigation on the effect of DFG-CNT materials on the nonlinear dynamic behavior of incomplete conical shells. CNTs are distributed with three different patterns along the thickness of the conical shell. In order to analyze the dynamic of this system, first, the nonlinear dynamic equations of the conical shell are derived based on the first-order shear deformation theory and van Karman's strain-displacement relations, and then they are solved using the Galerkin method and the Adams-Bashforth numerical method. Finally, the effects of power index, volume fraction and distribution pattern of CNTs, geometric characteristics of the shell, frequency and amplitude of the excitation force on the nonlinear dynamic response of the conical shell are investigated.

2. Geometrical Modeling

Figure 1 shows the schematic of a truncated conical shell reinforced with CNTs with ceramic and metal matrix material with length L, small radius R_1, large radius R_2, thickness h and cone angle α. The origin of cylindrical coordinates (x, θ, z) is located on the axis passing through the middle layer of the shell and on point O. In addition, Figure 1 shows that the matrix material of the shell is composed of two phases, metal and ceramic, whose properties change according to the thickness of the shell as a FGM. The metal part is shown in blue color and the ceramic part is shown in white color. Also, CNTs are distributed inside the base material along the thickness of the shell.

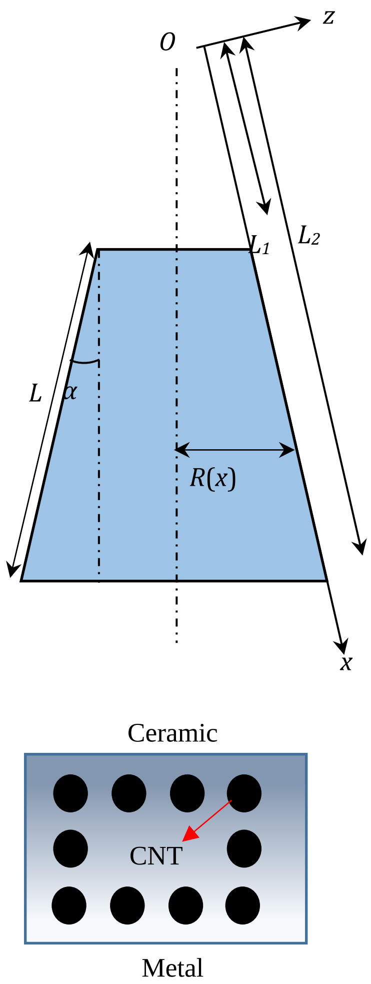

Figure 1. Schematic of truncated conical shell reinforced with CNTs with ceramic and metal matrix

3. Mechanical Properties

The studied composite conical shell consists of two phases of base material and reinforcement, the base material (matrix) is a combination of ceramic and metal, whose properties vary according to the thickness of the shell. Carbon nanotubes play the role of reinforcement, which are injected into the base material. The distribution of carbon nanotubes along the thickness of the shell is functionally graded. It should be noted that in this research, four different types of functions have been considered for the distribution pattern of carbon nanotubes along the thickness of the shell. According to Figure 2, four types of functions with UD, FGV, FGO and FGX distributions have been considered for the distribution of CNTs along the shell thickness.

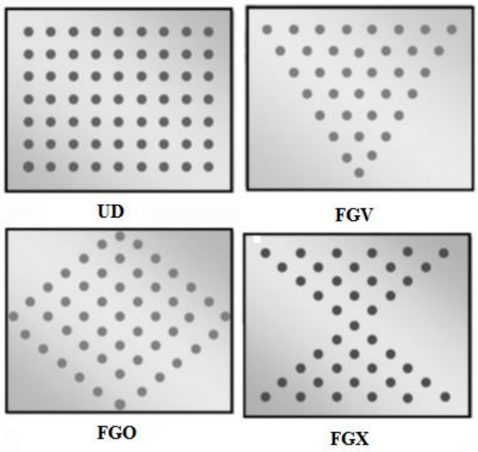

Figure 2. Types of distribution patterns of CNTs

4. Governing Equations

For the dynamic analysis of the system, the dynamic equations of the system must be obtained. For this purpose, using Hamilton's principle, first order shear deformation theory and von Karman's nonlinear strain-displacement relations, the nonlinear dynamic equations of the shell are obtained:

$$-\frac{\sin(\alpha)\,N_{\theta\theta}}{R(x)}+\frac{\frac{\partial N_{x\theta}}{\partial\theta}}{R(x)}+\frac{\sin(\alpha)\,N_{xx}}{R(x)}+\frac{\partial N_{xx}}{\partial x}=I_0\ddot{u}+I_1\ddot{\phi}_{xx} \tag{1}$$

$$\frac{\frac{\partial N_{\theta\theta}}{\partial\theta}}{R(x)}+\frac{2\sin(\alpha)N_{x\theta}}{R(x)}+\frac{\partial N_{x\theta}}{\partial x}+\frac{\cos(\alpha)Q_{\theta\theta}}{R(x)}=I_0\ddot{v}+I_1\ddot{\phi}_{\theta\theta} \tag{2}$$

$$\begin{aligned}\frac{2N_{x\theta}\frac{\partial^2 w}{\partial x\,\partial\theta}}{R(x)}+\frac{N_{\theta\theta}\frac{\partial^2 w}{\partial\theta^2}}{R(x)^2}+\frac{\frac{\partial w}{\partial\theta}\frac{\partial N_{\theta\theta}}{\partial\theta}}{R(x)^2}+\frac{\frac{\partial w}{\partial\theta}\frac{\partial N_{x\theta}}{\partial x}}{R(x)}&\\+\frac{\frac{\partial w}{\partial x}\frac{\partial N_{x\theta}}{\partial\theta}}{R(x)}+\frac{\sin(\alpha)N_{xx}\frac{\partial w}{\partial x}}{R(x)}&\\-\frac{\cos(\alpha)\,N_{\theta\theta}}{R(x)}+N_{xx}\frac{\partial^2 w}{\partial x^2}&\\+\frac{\partial w}{\partial x}\frac{\partial N_{xx}}{\partial x}+\frac{\frac{\partial Q_{\theta\theta}}{\partial\theta}}{R(x)}+\frac{\sin(\alpha)\,Q_{xx}}{R(x)}&\\+\frac{\partial Q_{xx}}{\partial x}+F=I_0\frac{\partial^2 w}{\partial t^2}&\end{aligned} \tag{3}$$

$$\frac{\frac{\partial M_{x\theta}}{\partial\theta}}{R(x)}+\frac{M_{xx}-M_{\theta\theta}}{x}+\frac{\partial M_{xx}}{\partial x}-Q_{xx}=I_1\ddot{u}+I_2\ddot{\phi}_{xx} \tag{4}$$

$$\frac{\frac{\partial M_{\theta\theta}}{\partial\theta}}{R(x)}+\frac{2\sin(\alpha)M_{x\theta}}{R(x)}+\frac{\partial M_{x\theta}}{\partial x}-Q_{\theta\theta}=I_1\ddot{v}+I_2\ddot{\phi}_{\theta\theta} \tag{5}$$

5. Galerkin Procedure

Galerkin method is used to convert partial differential equations into ordinary differential equations. For this purpose, first, the following mode shape functions are considered to satisfy the boundary conditions of the system:

$$u(x,\theta,t)=U(t)\cos\left(\frac{m\pi(x-L_1)}{L}\right)\cos(n\theta) \tag{6}$$

$$v(x,\theta,t)=V(t)\sin\left(\frac{m\pi(x-L_1)}{L}\right)\sin(n\theta) \tag{7}$$

$$w(x,\theta,t)=W(t)\sin\left(\frac{m\pi(x-L_1)}{L}\right)\cos(n\theta) \tag{8}$$

$$\phi_{xx}(x,\theta,t)=X(t)\cos\left(\frac{m\pi(x-L_1)}{L}\right)\cos(n\theta) \tag{9}$$

$$\phi_{\theta\theta}(x,\theta,t)=Y(t)\sin\left(\frac{m\pi(x-L_1)}{L}\right)\sin(n\theta) \tag{10}$$

Substituting Eqs. (6) to (10) in Eqs. (1) to (5) and then applying the Galerkin method, the nonlinear partial differential equations of the system are converted into ordinary differential equations of the time function:

$$S_{11}(u)+S_{12}(v)+S_{13}(w)+S_{14}(\phi_{xx})+S_{15}(\phi_{\theta\theta})+P_1=I_0\ddot{u}+I_1\ddot{\phi}_{xx} \tag{11}$$

$$S_{21}(u)+S_{22}(v)+S_{23}(w)+S_{24}(\phi_{xx})+S_{25}(\phi_{\theta\theta})+P_2=I_0\ddot{v}+I_1\ddot{\phi}_{\theta\theta} \tag{12}$$

$$S_{31}(u)+S_{32}(v)+S_{33}(w)+S_{34}(\phi_{xx})+S_{35}(\phi_{\theta\theta})+P_3+F=I_0\ddot{w} \tag{13}$$

$$S_{41}(u)+S_{42}(v)+S_{43}(w)+S_{44}(\phi_{xx})+S_{45}(\phi_{\theta\theta})+P_4=I_1\ddot{u}+I_2\ddot{\phi}_{xx} \tag{14}$$

$$S_{51}(u)+S_{52}(v)+S_{53}(w)+S_{54}(\phi_{xx})+S_{55}(\phi_{\theta\theta})+P_5=I_1\ddot{v}+I_2\ddot{\phi}_{\theta\theta} \tag{15}$$

that L_{ij} and N_{ij} are linear and non-linear dimensionless coefficients, respectively, which depend on the system's gender and geometry. To calculate the natural frequencies of the conical shell, it is enough to first ignore the nonlinear terms of Eqs. (11) to (15) and then calculate the following matrix's determinant:

$$\begin{bmatrix}L_{11}+I_0\omega^2 & L_{12} & L_{13} & L_{14}+I_1\omega^2 & L_{15}\\ L_{21} & L_{22}+I_0\omega^2 & L_{23} & L_{24} & L_{25}+I_1\omega^2\\ L_{31} & L_{32} & L_{33}+I_0\omega^2 & L_{34} & L_{35}\\ L_{41}+I_1\omega^2 & L_{42} & L_{43} & L_{44}+I_2\omega^2 & L_{45}\\ L_{51} & L_{52}+I_1\omega^2 & L_{53} & L_{54} & L_{55}+I_2\omega^2\end{bmatrix} \tag{16}$$

6. Adams-Bashforth Method

In this part, the Adams-Bashforth numerical method is used to calculate the nonlinear dynamic response of the shell under sinusoidal harmonic loading. In other words, the system of Eqs. (1) to (5) is solved with the help of this method. The Adams-Bashforth method is an explicit method that approximates the solution of an ordinary differential equation by extrapolating from previous values. However, it can be extended to solve time-dependent partial differential equations using spatial discretization techniques such as finite difference or finite element methods.

7. Results and Discussion

After ensuring the correctness of the results, in this part, the effect of different parameters of the system on the nonlinear dynamic response of the system is investigated. Figure 3 shows the diagram of the nonlinear dynamic response of the conical shell for different values of the power index. As it is clear from this diagram, with the increase of the power index, the natural frequency of the shell decreases. In addition, it is clear from these graphs that with the increase of the power index, the amplitude of vibrations of the shell increases independently of the distribution pattern of CNTs.

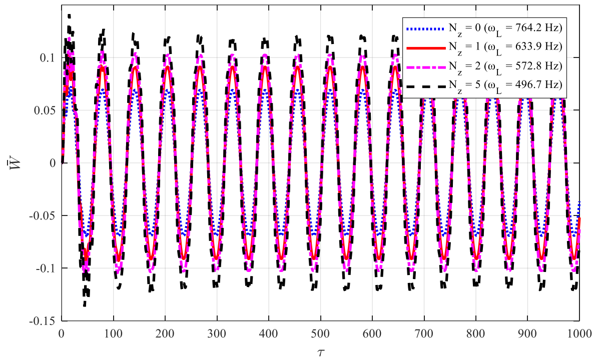

Figure 3. Effect of power index on nonlinear dynamic response of conical shell reinforced with CNTs

Figure 4 shows the diagram of the nonlinear dynamic response of the conical shell for different amounts of volume fraction of CNTs. As it is clear from this graph, with the increase in the volume fraction of CNTs, the natural frequency of the shell increases and the vibrations amplitude decreases.

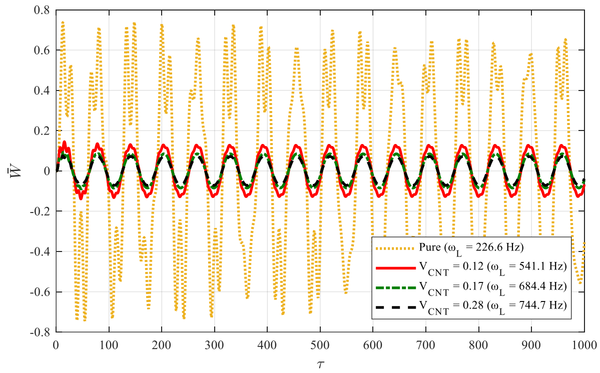

Figure 4. Effect of volume fraction of CNTs on nonlinear dynamic response of conical shell reinforced with CNTs

Figure 5 shows the non-linear dynamic response diagram of the conical shell for different types of CNTs distribution pattern. As it is clear from these graphs, the highest and lowest natural frequencies correspond to the distribution pattern of FGX and FGO, respectively. In addition, the highest and lowest amplitude of fluctuations are related to the distribution pattern of FGO and FGX, respectively.

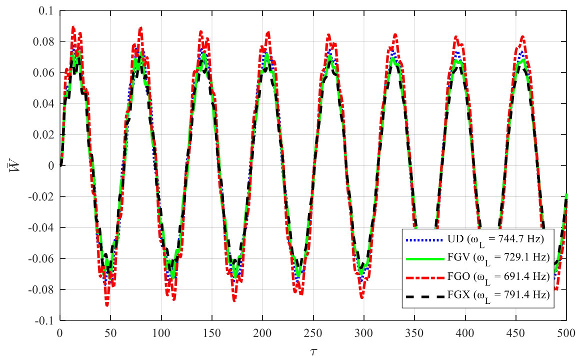

Figure 5. Distribution pattern of CNTs on nonlinear dynamic response of conical shell reinforced with CNTs

8. Conclusion

In this research, the analysis of the nonlinear dynamic response of conical shells reinforced with CNTs with a FG matrix has been done. For this purpose, the nonlinear dynamic equations of the conical shell were derived based on the first-order shear deformation theory and von Karman's strain-displacement relations and solved by the Adams-Bashforth numerical method. Then the effect of different parameters of the system on its dynamic response was investigated.

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