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Developing a Multi-Objective Game Theoretic Design of Path-Generating Planar Mechanism

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Abstract

This article addresses the bi-level multi-objective optimization problems raised in reliability-based robust design optimization of engineering applications through establishing a state-of-the-art game theoretic scenario. A novel bi-level decentralized decision-making approach is proposed using the synergy of reliability-based robust design optimization (RBRDO), Stackelberg/cooperative game theory, Monte Carlo simulation (MCS) and genetic programming (GP). The application of the proposed approach is elaborated in a case-study of multi-objective robust synthesis of high-speed path generating four-bar. The four performance criteria, namely, accuracy (TE), robustness (μ_{TE} and σ_{TE}^2), reliability (f_G) at upper level and quality of motion (TA) at lower level are assigned to four players so that each of whom is in charge of one objective criterion being optimized. The peak input driving torque (T_S) as the dynamic constraint is associated with the upper-level problem. The genetic programming (GP) metamodel is used to capture the Stackelberg protocol between two levels *i.e.,* constructing the follower's rational reaction set (RRS) and the Nash bargaining function is hired to model the cooperative behaviors in upper level. The obtained results show a considerable enhancement in reliability and robust behavior of mechanism.

Keywords: Multi-objective optimization; Game theory; robust design; Genetic programming; Path-generating four-bar.

1. Introduction

Optimal synthesis of four-bar linkages for path-generating applications has always attracted the curiosity of engineers and researchers in the field of dimensional design of planar mechanisms [1, 2]. In some recently papers, the game theoretic approaches are used to address the deterministically multi-objective design of mechanism. The authors in [3] elaborated and successfully implemented a novel approach using the synergy of Stackelberg game theory and GMDH-type neural network for two case studies of path synthesis of four-bar mechanisms. In another paper [4], the authors introduced a novel analytical method to establish a logical RRS for the follower. As a result, the multi-objective optimal synthesis of the four-bar mechanism is converted to a single-objective problem solely utilizing leader variables and the acquired nonlinear algebraic equation that represents the RRS. A synergetic approach on the basis of cooperative game and reliability-based design optimization to elaborate the design scenarios arising in path generating purpose of four-bars was proposed in [5].

In addition to accuracy, the designers usually confront some other kinematic performance criteria e.g., robustness, reliability and quality of motion. It is essential to address these important criteria to achieve an accurate and also robust, reliable and efficient mechanism that works satisfactorily during its period of life. Moreover, in order to enhance the overall synthesis of a high-speed four-bar linkage, both the dynamic and kinematic criteria are generally required to be involved at the same time.

This study addresses the problem of optimal synthesis of four-bars by elaborating a bi-level multi-objective game-based model in conjunction with RBRDO, GP and Monte Carlo simulation (MCS). The four performance criteria, namely, accuracy (TE), robustness (μ_{TE} and σ_{TE}^2), reliability (f_G) at leader level and quality of motion (TA) at follower level are associated with four players each of whom is in charge of one performance criterion to be optimized. In the suggested scheme, the three players in upper level cooperate with each other to construct a bargaining function which ensures the Pareto optimality condition. The Stackelberg game scenario is then utilized to model the inter-level behaviors of players. A synergetic approach of GP and MCS is used to establish the approximated model for the follower's RRS. In this way, the four-objective optimal synthesis of planar four-bar linkage is reduced into a single-objective robust path generating design problem.

2. Methodology

Figure 1 depicts a four-bar linkage for path generating application. The optimal path synthesis mechanism consists of evaluating the design parameters $\boldsymbol{X} = [r_1, r_2, r_3, r_4, r_{cx}, r_{cy}, \theta_0, x_0, y_0]$ wherein tracking error (TE) is minimized over some trajectory targets.

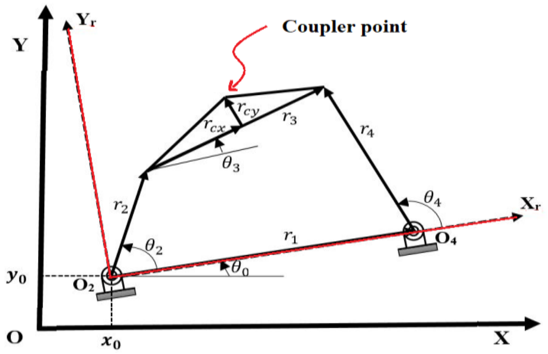

Figure 1. Four-bar mechanism

TE, which calculates the structural error of path generating mechanism, can be easily represented as

$$TE = \sum_{i=1}^{N} [(C_{xd}^{i} - C_{x}^{i})^2 + (C_{yd}^{i} - C_{y}^{i})^2] \, , \qquad (1)$$

where, $C_d^i = [C_{xd}^i,\ C_{yd}^i]^T$ and $C^i = [C_x^i,\ C_y^i]^T$ indicate respectively the set of desired and traced points. Readily, the accuracy of mechanism is defined through the minimization of TE. The objective function of transmission angle (TA) as a measure of quality of motion is formulated as

$$TA = [(\gamma_{max} - 90^o)^2 + (\gamma_{min} - 90^o)^2] \qquad (2)$$

in which, the minimum maximum values of γ is evaluated by

$$\gamma_{max} = \cos^{-1}\left[\frac{r_4^2 - (r_1 + r_2)^2 + r_3^2}{2 r_4 r_3}\right] \qquad (3a)$$

$$\gamma_{min} = \cos^{-1}\left[\frac{r_4^2 - (r_1 - r_2)^2 + r_3^2}{2 r_4 r_3}\right] \qquad (3b)$$

To achieve a robust mechanism, the mean of tracking error (μ_{TE}) and standard deviation of tracking error (σ_{TE}^2) are required to minimized so that sensitivity of the mechanism to its parameter's uncertainty is alleviated as much as possible . Moreover, guaranteeing Grashof condition under presence of uncertainty is the most of importance at both stages of design and manufacturing. Hence, it is required to define an objective function from RBDO perspective, namely, the failure probability of Grashof condition (f_G). In the present study, the peak input driving torque i.e., $\max(T_S) \leq 1$ is considered as the design constraint assigned to the upper level. Figure 2 shows the structure of the proposed game scenario in reliability based robust synthesis of high-speed four-bar linkage for path generating application. The three players 1, 2 and 3, namely, accuracy (TE), robustness (μ_{TE} and σ_{TE}^2) and reliability (f_G) are located at the leader level. The peak input torque over a cycle (T_S) is assigned as the constraint of the leader problem. The player 4, namely, quality of motion (TA) is placed in lower level and this level plays the role of the follower. The interaction between players 1, 2 and 3 at upper level is considered as a collaborative (Nash) game. The Nash bargaining function in Eq. (2) is hired to model the collaborative behavior among players. Stackelberg game theory is consequently utilized to model the behavior of upper and lower levels i.e., the GP-type metamodel is utilized to obtain an approximation to the RRS of the follower.

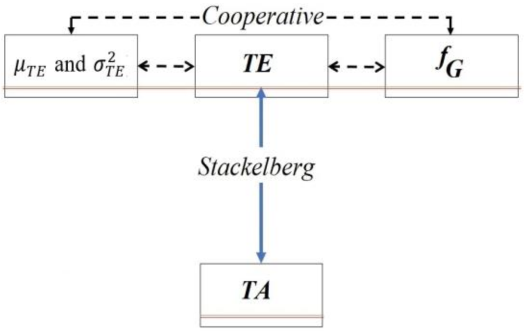

Figure 2. The proposed game theoretic scenario

The control of two design variables, r_3 and r_4 (which subtend the TA criterion) is associated with the follower, while $r_1, r_2, r_{cx}, r_{cy}, \theta_0, x_0, y_0$ are assigned to the leader. The bargaining functions at the upper level are constructed as follows

$$F_{B-leader} = f_{Accuracy} \times f_{Robustness} \times f_{Reliability} \qquad (4)$$

The four-objective optimization problem is now represented as below

$$\text{Upper level:} \quad \textbf{max} \ (F_{B-leader}) \quad \text{by varying } r_1, r_2, r_{cx}, r_{cy}, \theta_0, x_0, y_0 \qquad (5a)$$

$$\text{Lower level:} \quad \textbf{min} \ (TA) \quad \text{by varying } r_3, r_4 \qquad (5b)$$

A Stackelberg game-theoretic concept using synergy of GP and MCS is presented to find the follower's RRS level. The mathematical formulation of GP- type model for optimum r_3 and r_4 are given by (6). The proposed GP model impressively provides a high correlation coefficient of $R^2 = 0.98$. Finally, the leader's objective function is optimized using a single-objective optimization of (5).

$$r_3 = r_4 = \frac{r_1 + r_2}{1.56} \tag{6}$$

3. Results and Discussion

For the purpose of involving the uncertainty, this study considers normal distribution with ± 3 % of the nominal values of r_1, r_2, r_3 and r_4. However, the proposed approach is applicable for any amount of uncertainty with normal distribution. To implement the optimization of leader a MCS with 200 samples along with a real-coded genetic algorithm, a population of 60 with a mutation probability of 0.05 and crossover probability of 0.9 has been implemented in 200 iterations. In case-study, the set of desired points;

$$\{C_d^i\} = \left\{ \begin{array}{c} (20, 10), (17/66, 15/14), (11/73, 17/87), \\ (5, 16/92), (0/603, 12/73), (0/603, 7/26), \\ (5, 3/07), (11/73, 2/12), (17/66, 4/85), \\ (20, 10) \end{array} \right\}$$

specifies the desired path that is required to be traced by coupler point. Moreover, the vector of design parameters is $X = [r_1, r_2, r_3, r_4, r_{cx}, r_{cy}, \theta_0, x_0, y_0]$ and its range is set to $r_1, r_2, r_3, r_4 \in [0, 60], r_{cx}, r_{cy}, x_0, y_0 \in [-60, 60]$, and $\theta_0 \in [0, 2\pi]$. Table 1 shows the optimal design parameters of mechanisms, the deterministic values of objective criteria (which are evaluated at the nominal values of design parameters) as well as the statistical metrics of objective criteria of this work together with those reported in literature. From Table 1, it is evident that the game-based design of this research is considerably superior than the mechanism proposed in literature when considering the statistical objective functions μ_{TE}, σ_{TE}^2 and f_G. Interestingly, the values of TE and TA of the mechanisms of this work are better to other mechanism which is obtained on the basis of nominal values of mechanism dimension. Quantitatively, the results show a notable enhancement in reliability and robust behavior of mechanism, whilst both criteria of accuracy and motion quality is preserved.

Table 1. Comparative results of four-bar mechanisms

Mechanism / Parameter	**[1]**	This work
r_1	54.3608	54.9767
r_2	8.6833	9.4325
r_3	34.3186	45.5442
r_4	79.9961	45.5442
r_{cx}	0.00018	6.4483
r_{cy}	1.4652	6.6743
x_0	10.9543	2.2910
y_0	11.0745	4.9775
θ_0	2.1296	5.1520
TE	1.952	0.048
TA	9777.18	900
μ_{TE}	37.531	1.067
σ_{TE}^2	4.056	1.701
f_G (%)	48	0
T_S	3.6	1

Figure 3 shows the structure of the obtained four-bar linkage using the method of proposed bi-level multi-objective game scenario.

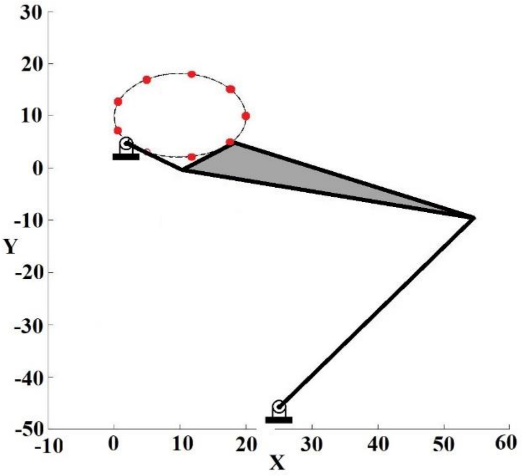

Figure 3. The structure of designed four-bar linkage

4. Conclusions

This article, proposed a novel synergetic approach based on decentralized hierarchical game theory and genetic programming to address the optimization of engineering applications. The reliability-based robust design optimization of high-speed path generating four-bar linkages was considered. In this way, four kinematic performance criteria, namely, accuracy, robustness, reliability and quality of motion were assigns to four players each of whom is in charge of one criterion to be optimized. Moreover, the peak input driving torque was involved as a dynamic design constraint. The obtained results show a considerable improvement in robust and reliability behavior of mechanism, whilst both performance criteria of accuracy and quality of motion in deterministic enouncement is preserved.

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