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Identification of the Failure Load of a Hyperelastic Body Considering the Location of the Failure

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Abstract

In recent years, analysis of inverse hyperelastic problems has received more attention than before. In this article, an inverse problem related to the failure of hyperelastic bodies is defined and two different methods are proposed to solve the problem. The inverse analysis of hyperelastic bodies that have failed can be useful to prevent the recurrence of failure in these materials. In the inverse problem, it is assumed that a two-dimensional hyperelastic solid is failed and the place of its failure is known. The distribution of the load (boundary conditions) in a part of the boundary is considered unknown and is calculated by solving the inverse problem. By defining an appropriate objective function, the inverse problem is converted to an unconstrained optimization problem. To solve the optimization problem, a zero-order method based on the equal interval search method and a first-order method based on the steepest descent method are used. To make the problem more practical, the inverse problem input data, which are the location of failure and the critical equivalent strain, are used with some error. It is observed that the performance of the first-order method is better than the zero-order method.

Keywords: Failure, Gradient-based, Hyperelastic, Inverse problem, Optimization.

1. Introduction

In the mechanical analysis of hyperelastic materials, both the material behavior and the deformation of the material are nonlinear. Optimization and inverse analysis of hyperelastic materials can help to improve some industrial equipment and can be useful in biomechanics fields such as heart valve construction, soft tissue repair, and prosthesis manufacturing. One of the most important properties of hyperelastic materials is that, they experience very large elastic strains under small stresses. Due to the non-linear behavior of hyperelastic materials, these materials do not obey Hooke's law, and for their analysis, large elastic deformation theories should be used.

So far, many studies have been conducted on the identification and modeling of hyperelastic materials. In 1997, Olusanya [1] presented a measurement method for the tensile failure of hyperelastic materials and extended it to viscoelastic and viscoplastic materials. He used strain energy density function of a specific material at seven different temperatures and after drawing several graphs, he obtained the relationship between failure energy and temperature. Volokh [2] presented a model for the failure of rubber-like materials and suggested a new failure potential function. Moreover, in 2011 Volokh [3] continued his research on the failure of hyperelastic materials and modeled the failure of soft non-isotropic materials. In another research conducted in 2020 by Hajhashemkhani et al. [4], a new inverse method was presented to identify the initial configuration of a deformed hyperelastic body using the finite element method. Recently, Zochowski et al. [5], in order to analyze the energy absorption and dissipation capabilities of hyperelastic materials, conducted experimental and numerical studies on the failure mechanisms of hyperelastic materials under impact loads.

According to previous investigations, it can be seen that extensive research has been carried out on hyperelastic materials and various inverse problems have been presented regarding the identification of material parameters and boundary conditions of hyperelastic materials. However, to the authors’ best knowledge, no work on identification of the failure loads of hyperelastic members is presented. In this work, a zero-order method based on the equal interval search method and a first-order method based on the steepest descent method are presented to solve this new kind of inverse problem.

2. Methodology

In this research, a homogenous and isotropic hyperelastic body is considered and it is assumed that, a failure has occurred at a known location of the body due to the application of a static load. The supports of the body, the material properties, and the location of the applied load are also known, but the distribution of the applied load that caused the failure is completely or partially unknown. The load is modeled in terms of two parameters (q_1 and q_2), which are the unknowns of the inverse problem. We consider the unknown parameters expressing the load in the form of the column matrix **q** as follows:

$$\mathbf{q} = \begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix} \quad (1)$$

By defining a suitable objective function, the inverse problem is converted to an unconstrained optimization problem. For this purpose, the following two-part objective function is considered:

$$g(\mathbf{q}) = \frac{(x_c(\mathbf{q}) - x_f)^2 + (y_c(\mathbf{q}) - y_f)^2}{x_f^{\,2} + y_f^{\,2}} + \lambda \frac{(\varepsilon_c(\mathbf{q}) - \varepsilon_f)^2}{\varepsilon_f^{\,2}} = g_1 + \lambda g_2 \quad (2)$$

The first part in the above equation is related to the location of the failure and the second part is related to the critical equivalent strain that occurs at the point of failure. We consider the coordinates of the failure point as (x_f, y_f), which is known, and we indicate the coordinates of the critical point with (x_c, y_c). ε_c is the value of ε_{eq} at the critical point, which is calculated by the software at each step, and ε_f is the amount of equivalent strain that causes failure, and its value is known. The coefficient λ in Eq. (2) expresses the weight of the function g_2 compared to g_1.

To solve the defined optimization problem, a zero-order method based on the equal interval search method and a first-order method based on the steepest descent method are used. When the zero-order method with the objective function given in Eq. (2) is used, the convergence process is slow and sometimes convergence to the solution does not occur. Therefore, the optimization problem is converted into two one-dimensional optimization problems.

There are many search methods to find the optimum point of a function. One of the simplest methods is the equal interval search method [6], which is used in this work.

The problem is also solved using a first-order optimization method. In a gradient-based (first-order) method, as the name suggests, the gradient and the derivative of the objective function is used to solve the optimization problem. The gradient-based method, which is used in this work, is the steepest descent method [6].

3. Discussion and Results

We consider a square hyperelastic pate with a corner cut-out under plane stress condition as shown in Figure 1. The dimensions of the plate and the radius of the quarter circle are as follows:

$$OD = OB = 0.5\,\text{m} \quad (3)$$

$$OF = 0.2\,\text{m} \quad (4)$$

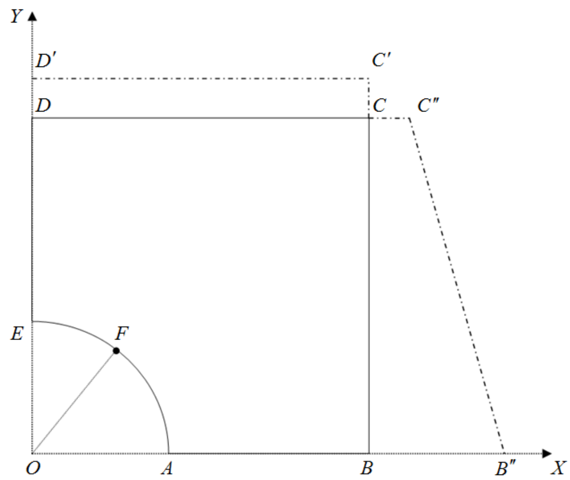

Figure 1. A hyperelastic plate with a corner cut-out

We consider a uniform vertical displacement for the upper edge *DC* and a horizontal displacement with linear variation for the right edge as follows:

$$v_{DC} = 0.1\,\text{m} \quad (5)$$

$$q_1 = u_C = 0.02\,\text{m} \quad (6)$$

$$q_2 = u_B = 0.12\,\text{m} \quad (7)$$

After analyzing the direct problem using the ANSYS software [7] and performing the necessary calculations to determine the equivalent strain at all the nodes by the MATLAB software, it can be seen that the maximum equivalent strain with the value of 0.5214 occurs at point *F* with coordinates (0.1541, 0.1275). These values are considered as the input data for the inverse problem, which is defined as follows:

In the plate shown in Figure 1, the material and boundary conditions on all edges except the edge *BC* are known (as before). The displacement of the edge *BC* is assumed horizontal with unknown linear variation. The location of failure is (0.1541, 0.1275) and the failure equivalent strain is 0.5214. The unknowns of the inverse problem, i.e. u_B and u_C, are considered as q_1 and q_2, respectively.

In order to make the research more practical, the input data of the inverse problem, which are the location of the failure and the failure equivalent strain, are used with some error. The obtained results can be seen in Tables 1 and 2.

Table 1. The results of the zero-order method with a measurement error of 3.5% in the coordinates of the failure location and 2% in the value of ε_f

$\sqrt{g_2}$	$\sqrt{g_1}$	u_C (m)	u_B (m)	The total number of steps	initial guess u_C (m)	initial guess u_B (m)
1.5318×10^{-7}	0	0.0112	0.1102	68	0.04	0.14

Table 2. The results of the gradient-based method with a measurement error of 3.5% in the coordinates of the failure location and 2% in the value of ε_f

$\sqrt{g_2}$	$\sqrt{g_1}$	u_C (m)	u_B (m)	The total number of steps	initial guess u_C (m)	initial guess u_B (m)
3.447×10^{-5}	1.011×10^{-3}	0.0285	0.1109	15	0.04	0.14

The results obtained from the zero-order method and the first-order method show that both methods have the ability to identify the unknown boundary condition. The accuracy of both methods is of the same order, but the number of steps required to solve the problem by the first-order method is less than that in the zero-order method.

4. Conclusions

In this article, an inverse method for determining the failure load of hyperelastic bodies using the location of the failure was presented. According to the presented examples, it could be seen that both the zero-order and first-order methods have the ability to solve the considered problem, but the performance of the first-order method is much better than the zero-order method.

In general, it can be seen that the gradient-based method, despite the fact that the equations used in it are more complicated than the zero-order method, gives relatively better results in a shorter time, and this means that in solving the inverse problem of this research, the gradient-based method is recommended.

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