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Investigating the influence of bearings elastic properties on nonlinear dynamics of unbalanced induction motors shaft in the presence of asymmetric magnetic pull

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Abstract
This paper aims to investigate the influence of bearing elastic properties on nonlinear dynamics of unbalanced rotors. Accounting to the influence of asymmetric magnetic pull, the governing equations of motion associated with the rotor are obtained using the nonlinear Euler-Bernoulli beam theory. Adopting the Galerkin projection method, the reduced equations of motion are extracted and then solved analytically through the method of multiple time scales for the cases of free vibrations and primary resonances. Aside from the numerical simulations, the present findings are compared and successfully validated by those published in the previous studies. Afterward, a detailed parametric study is conducted to assess the influences of asymmetric magnetic pull, nonlinear stiffnesses of the bearings and the eccentricity on nonlinear dynamics of the system. Results reveal that accounting for the influence of asymmetric magnetic pull decreases the natural frequencies of the system. In addition, it is observed that increasing the eccentricity increases the amplitudes of vibrations and also broadens the bi-stable resonance zone.
Keywords: Nonlinear dynamics; Induction motors shaft; Asymmetric magnetic pull; Nonlinear stiffnesses of bearings; Eccentricity.

1. Introduction

Nonlinear dynamical analysis of induction electric motors in order to shedding light on its working condition is of great interest nowadays. So, it is very important to model the system as close as possible to the real conditions in which it operates. Here the pioneering literatures devoted to modelling dynamical behavior of rotating shafts are reviewed.

Neglecting the effect of shaft's mid-plane stretching, Hosseini and Khadem [1] analytically analyzed nonlinear free vibrations of rotors. They showed that in real rotors, there is no practical stretching for the middle surface, and so the nonlinearity of the system in the whirling mode is due to the nonlinearity of the rotating shaft curvature and inertia. Lu and Wang [2] numerically investigated the nonlinear vibrations of rotors under electromagnetic excitation by ignoring gyroscopic effects and geometric nonlinearity. They discretized the governing equations of motion using Galerkin's method and solved the resulting initial value problems by the Runge-Kutta procedure. Accounting for the effects of nonlinear curvature and inertia, Eftekhari et al. [3] studied the effect of stator magnetic field on the vibrations of induction motors shafts in presence of unbalance mass. It is worth mentioning that in all the above studies, the bearings were considered rigid. In order to investigate the effects of the elastic properties of the bearings, Phadatare and Pratiher [4] modeled bearings as springs with nonlinear stiffnesses and studied nonlinear vibrations of the rotor resting on such supports. However, the combined influences of nonlinear elastic bearings and stator magnetic pull on the nonlinear dynamical behavior of the system has not been studied yet. Therefore, the current work focuses on the nonlinear analysis of such a system via a large deformable model.

2. Mathematical formulation of the governing equations of motion

2.1. *Rotor-Bearing modeling*

Figure 1 illustrates a schematic of an elastic rotor as a beam of length *L* and mass *M* resting on elastic supports at its both ends. To obtain the governing equations of motion, a body-fixed coordinate system *XYZ* which is attached to the mid-surface of the rotor on its left hand side and rotates with it, is utilized. In addition, a local coordinate system $\xi\eta\zeta$ which is attached to the mid-surface of the shaft at a given cross-section so that the ξ-axis is always being perpendicular to that cross section, is also employed. The bearings are also modeled as a set of linear and non-linear springs along with viscus dampers.

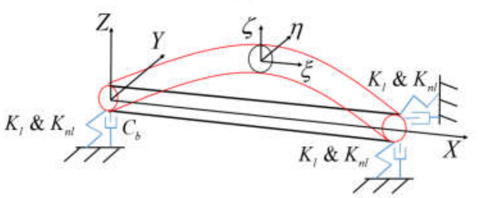

Figure 1. Schematic of a rotor resting on elastic supports.

According to Figure 2, O' and O are the centers of the rotor and stator, respectively. As can be seen in Figure 2, these two centers do not coincide each other. The distance between the centers of the rotor and the stator is determined by the parameter *e*.

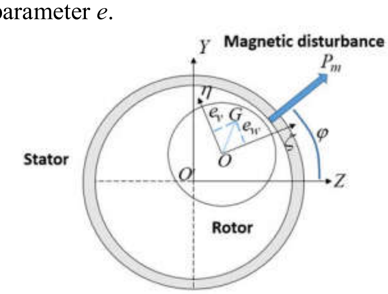

Figure 2- Representation of rotor and stator centers in presence of the magnetic excitation

As Figure 2 demonstrates, the mass center of the rotor (i.e. *G*) also does not coincide with its geometric center (i.e.O') and is defined by two components e_v ande_w. In addition, the rotor cross-section twist angle is determined by φ. Using Euler's *zyx* consecutive rotations, it is possible to relate the global and local reference frames to each other. That is the *XYZ* reference frame is firstly rotated around its *Z* axis by the angle ψ to obtain the X_1Y_1Z reference frame, then the X_1Y_1Z reference frame is rotated around its Y_1axis by the angle θ to obtain the $X_2Y_1Z_2$ reference frame, and finally the $X_2Y_1Z_2$ reference frame is rotated around its X_2 axis by the angle φ to reach the local $\xi\eta\zeta$ reference frame. Using Eq. (1), the R_{zyx} rotation matrix transforms the local $\xi\eta\zeta$ reference frame to the global *XYZ* coordinate system [5].

$$R_{zyx} = \begin{bmatrix} c\psi c\theta & c\psi s\theta s\varphi - s\psi c\varphi & c\psi s\theta c\varphi + s\psi s\varphi \\ s\varphi c\theta & -s\psi s\theta s\varphi + c\psi c\varphi & -s\psi s\theta c\varphi - c\psi s\varphi \\ -s\theta & c\theta s\varphi & c\theta c\varphi \end{bmatrix} \quad (1)$$

The displacements of a point on the cross section of the rotor along the *X, Y* and *Z* axes are expressed by the variables *u(s, t), v(s, t)* and *w(s, t),* respectively. The parameter *s* is the curvature length. Assuming the shaft mid-plane is inextensible and no shear deformation exists, the Euler angles ψ and θ can be expressed according to Eq. (2) in terms of the spatial derivative of the variables *v* and *w* [3].

$$\psi = tan^{-1}\left(\frac{\acute{v}}{1+\acute{u}}\right), \quad \theta = tan^{-1}\left(-\frac{\acute{w}}{(1+\acute{u})^2+\acute{w}}\right) \quad (2)$$

2.2. *Electromagnetic force*

Due to the non-uniform distribution of the magnetic field in the space between the rotor and the stator, the electromagnetic force acts on the system as an external excitation according to Eq. (3) [3].

$$P_m = \lambda_m e\left(\frac{1-e^2}{{C_e}^2}\right)^{-\frac{3}{2}} \quad (3)$$

Where the electromagnetic parameter λ_m is given by

$$\lambda_m = \frac{\pi {B_0}^2 R}{\mu_0 C_e} \quad (4)$$

2.3. *Elastic properties of the bearings*

Since the bearings are modeled as springs and dampers in the present study, the spring and damper forces are given Eqs. (5) and (6), respectively. These forces are the same for both *Y* and *Z* directions:

$$F_{spring} = \begin{pmatrix} K_l v + K_{nl} v^3 \\ +K_l w + K_{nl} w^3 \end{pmatrix} \delta(x) + \begin{pmatrix} K_l v + K_{nl} v^3 \\ +K_l w + K_{nl} w^3 \end{pmatrix} \delta(x - L). \quad (5)$$

$$F_{damping} = (C + C_b)(\dot{v} + \dot{w})\delta(x) + (C + C_b)(\dot{v} + \dot{w})\delta(x - L). \quad (6)$$

2.4. *Equations of motion*

According to Hamilton's principle, one can write:

$$\int_{t_1}^{t_2} \delta H \, dt = \int_{t_1}^{t_2} \delta(L + W_{ex}) \, dt = 0 \quad (7)$$

Where L is the Lagrangian and W_{ex} is the work done by the external damping and electromagnetic forces. Neglecting the influence of the gravity [3, 6], Eq. (7) can be simplified to:

$$\delta H = \int_{t_1}^{t_2} \begin{pmatrix} \delta T_{tr} + \delta T_{ro} - \delta U_{strain} - \delta U_{spring} \\ +\delta G^* + \delta W_{ex} \end{pmatrix} = 0 \quad (8)$$

where T_{ro} is rotational kinetic energy, T_{tr} is lateral kinetic energy, U_{strain} is the strain energy, U_{spring} is elastic potential energy stored in the bearings, G^* is the constraint equation comes from the inextensibility condition and W_{ex} is the work done by the external forces. By integrating Eq. (8) by parts and using the fundamental lemma of variational calculus, one can simply reach the governing equations of motion.

3. The method of multiple time scales

Herein, according to the Galerkin method, the variable *v* and *w* are discretized as:

$$v(s.t) = q_v(t)\phi(s) \quad (9)$$

$$w(s.t) = q_w(t)\phi(s) \quad (10)$$

Substituting Eqs. (9) and (10) in to the lateral and transvers governing equations of motion, multiplying both sides of these two equations by $\phi(s)$ and integrating the outcome over the whole region, the reduced nonlinear equations of motion will be obtained.

The method of multiple time scales is one of the analytical methods that can solve nonlinear initial value problems by introducing time scale $T_n = \varepsilon^n t$. In view of the introduced time scales, the time derivatives take the forms of [7]:

$$\frac{d}{dt} = D_0 + \varepsilon D_1 + \varepsilon^2 D_2 + \cdots \quad (11)$$

$$\frac{d^2}{dt^2} = {D_0}^2 + 2\varepsilon D_0 D_2 + \cdots \quad (12)$$

Where $D_n = \frac{\partial}{\partial T_n}$ $(n = 0, 1, 2, \dots)$ and ε is the book keeping parameter. Expanding the generalized coordinates q_v and q_w in the form a power series with odd exponents with respect to the book keeping parameters, substituting them into the reduced equations of motion and collecting the terms with like powers of ε, the forward and backward frequencies can be obtained by vanishing the determinant of the coefficients of the resulting eigenvalue problem.

Following the method of multiple times scales, mentioned above, for the case of primary resonance in which the excitation frequency leads to the resonance frequency of the system, the frequency response of the system can also be determined.

4. Result

4.1 *validation*

Table 1 provides the parameters utilized in validating the present findings. Figure 3 compares the frequency response of the present study with that reported in ref. [4].

Table1: Parameters utilized in Figure 3

Parameter	Value
I_1	0.000625
C	0.05 N-s/m
L	1m

According to Figure 3, it can be observed that the present results math exactly with those published in the literature. In addition, as is seen, the present system has a hardening behavior.

4.2. *Results and discussions*

Figure 4 presents the variation of the forward and backward frequencies versus the rotational speed of the rotor accounting for the influence of the electromagnetic parameter. As is seen, increasing the electromagnetic parameter decreases both the forward and backward frequencies of the system.

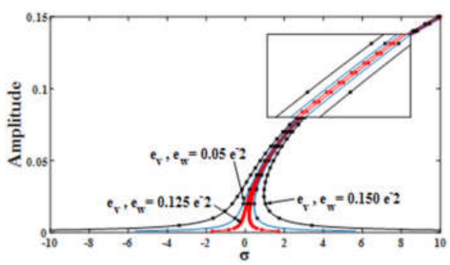

Figure 3- Comparison between the present frequency response curves (markers) and those reported in ref. [4] (solid lines) for a system whose properties are given in Table 1.

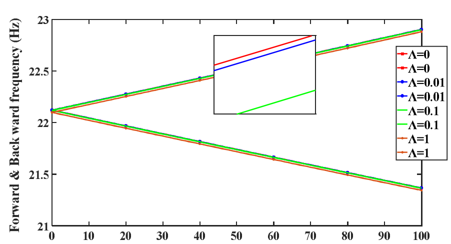

Figure 4- Campbell diagram of the present system accounting for the influence of electromagnetic pull.

Figure 5 illustrates the frequency responses of the system under the combined influences of electromagnetic pull, nonlinear stiffness of the bearings, eccentricity and rotor inertia. As is seen, increasing the non-linear stiffness of the bearings or reducing the rotor inertia increases the hardening behavior of the system. Also, increasing the electromagnetic parameter or reducing the eccentricity constricts the bi-stable zone of the frequency response curve.

5- Conclusion

In the present study, the governing equations of motion of the system have been derived using Hamilton's principle and reduced through the Galerkin method. The reduced equations of motion have then been solved analytically using the method of multiple time scales. The present findings have been comparted and successfully validated by those available in the literature. A detailed parametric study was then performed to illustrate the combined effects of nonlinear bearing stiffnesses, magnetic pull and eccentricity on nonlinear dynamics of the system.

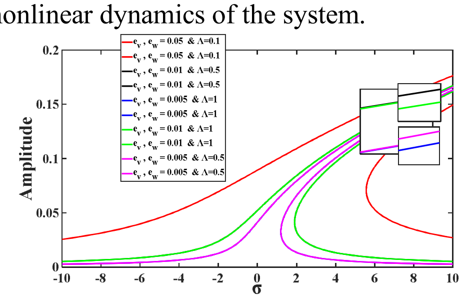

Figure 5- Frequency response of the system under the combined influences of the non-linear bearing stiffness, magnetic pull, eccentricity and inertia.

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