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Investigating the effects of cross section asymmetry on the behavior of the nonlinear nanoresonator under internal resonance conditions

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Abstract

As resonant micro/nanoresonators are very delicate devices with very small dimensions, therefore, any defects and faults caused by the process of manufacturing and laboratory implementation can lead to fundamental changes in their vibration behaviors. Therefore, the effects of the mentioned disadvantages should be considered as much as possible to obtain more accurate sensors with higher efficiency. In this study, a general model of a doubly clamped microbeam (nanotube) with asymmetric cross-section with external excitation is examined. Then, linear and non-linear behaviors of an ideal nanotube with circular cross section are investigated. The results of the simulations indicate good agreement with the experimental references available in the literature. Then, taking into account the asymmetry in the resonator cross-section, the system is moved away from an ideal model to a more real model, and the possible effects of the asymmetric cross-section in adjustment, reduction, and vanish of internal resonance are investigated and studied. Finally, the advantages and disadvantages caused by asymmetries and the optimal use of such an opportunity to obtain more innovative and complete models with higher efficiency are explained and detailed.

Keywords: Nonlinear micro/nano-mechanical resonators, intermodal coupling, internal resonance, Multiple Scales perturbation method, asymmetric resonator.

1. Introduction

Recently, M/NEMS resonators have received a lot of attention due to low mass, nanometer size, frequency tunability, ultrahigh frequency, and high Q-factor. They are used to detect physical quantities such as temperature[1], molecular masses[2], and force in the zeptoNewton level[3]. Due to their small size and very low mass, nanoresonators easily exhibit nonlinear behavior. One of the interesting phenomena in the nonlinear region, which strongly affects the modal response of NEMZ resonators and leads to a change in their resonance frequencies and quality factor, is nonlinear modal coupling. Strong modal coupling or internal resonance (IR) happens in NEMS resonators when the ratio between resonance frequencies of the coupled modes is an integer n or a nearly integer. So far, internal resonance has been studied in various structures such as cantilevers[4], curved beams[5], membranes[6], and carbon nanotubes[7]. The first experimental implementation of internal resonance with a frequency ratio of 1:3 between bending and transverse modes was realized in a doubly clamped microbeam under electrostatic excitation, which was used as a mechanism for frequency stabilization[8]. Similarly, in another study, it was observed when the excitation force is off, the amplitude of bending vibrations remained constant for a finite period of time because of energy exchange between bending and torsional modes in the internal resonance conditions[9]. Therefore, nonlinear modal coupling, especially internal resonance, can be used as a useful platform for engineering scenarios and energy harvesting in nanoresonators. In the context of linear resonators, imperfections and/or geometric asymmetry caused by the fabrication process and laboratory implementation have always been discussed and studied, and researchers have always looked for solutions to optimally exploit such challenges[10,11]. The mode coupling induced by geometric asymmetry in microcantilevers leads to enhance mass sensitivities and the imaging quality[12]. Recently, an extensive and up-to-date review is published, including a categorization of micro/nanoresonators based on linear/nonlinear, single/array, symmetric/asymmetric, along with frequency shift-based and amplitude shift-based resonators[13].

However, defects and faults caused by the process of manufacturing and laboratory implementation in nonlinear nanoresonators under internal resonance condition have received less attention until now.

In this study, first, a general model of a doubly clamped microbeam (nanotube) with external excitation is considered. In this regard, first the equations of continuous system are converted to nonlinear reduced-order equations with two degrees of freedom through the Gelerkin method. Then the resulting nonlinear equations are solved using the Multiple Scales perturbation method for an ideal nanotube with circular cross section and its linear and non-linear behaviors are investigated.

Then, taking into account the asymmetry in the resonator cross-section, the system is moved away from an ideal model to a more real model, which can be modeled as an elliptical cross-section with two main inertial axes, and the possible effects of the asymmetric cross-section in adjustment, reduction, and vanish of internal resonance are investigated and studied. Finally, the advantages and disadvantages caused by asymmetries and the optimal use of such an opportunity to obtain more innovative and complete models with higher efficiency are explained and detailed.

2. Problem formulation

Transverse vibrations of a model of a doubly clamped microbeam (nanotube) with asymmetric cross-section with external excitation are governed by generalizing integral-partial differential equation [14]:

$$\begin{aligned} &W_{,\tau\tau}+\frac{1}{Q}W_{,\tau}+\frac{1}{\beta^4}W_{,XXXX} \\ &-\frac{1}{2R_1^2\beta^4}W_{,XX}\int_0^1\left[W_{,X}^2+V_{,X}^2\right]dX \\ &=F(X,\tau) \\ &V_{,\tau\tau}+\frac{1}{Q}V_{,\tau}+\frac{1}{\beta^4}V_{,XXXX} \\ &-\frac{1}{2R_2^2\beta^4}V_{,XX}\int_0^1\left[W_{,X}^2+V_{,X}^2\right]dX \\ &=0 \end{aligned} \quad (1)$$

where the nondimensional parameters as follows

$$W=\frac{w}{h},\qquad V=\frac{v}{h},\qquad D=\frac{d_2}{h},\qquad X=\frac{x}{L},\qquad R_1^2=\frac{I_{zz}}{Ah^2},\qquad R_2^2=\frac{I_{yy}}{Ah^2},\qquad \tau=\omega_0 t \quad (2)$$

By using the Gelerkin method, nonlinear reduced-order equations with two degrees of freedom are obtained as follows.

$$\ddot{u}_1+{\omega_1}^2u_1=\frac{-1}{Q}\dot{u}_1-8\alpha_1u_1^3-8\alpha_1u_1u_2^2-2F_0\cos\Omega t \quad (3)$$

$$\ddot{u}_2+{\omega_2}^2u_2=\frac{-1}{Q}\dot{u}_2-8\alpha_2u_2^3-8\alpha_2u_2u_1^2 \quad (4)$$

Equations (3) and (4) can be solved by using the Multiple Scales perturbation method.

$$\begin{aligned} &T_0=t,\qquad T_1=\varepsilon t,\qquad T_2=\varepsilon^2 t\qquad D_n=\frac{\partial}{\partial T_n} \\ &u_1=\varepsilon u_{11}(T_0,T_2)+\varepsilon^3u_{13}(T_0,T_2)+\cdots \\ &u_2=\varepsilon u_{21}(T_0,T_2)+\varepsilon^3u_{23}(T_0,T_2)+\cdots \end{aligned} \quad (5)$$

In an ideal nanoresonator with circular cross section, internal resonance (IR=1:1) is realized because $\omega_1=\omega_2=\omega_0$.

Under internal resonance condition, detuning parameters σ_1 and σ_2 are introduced as follows.

$$\begin{aligned} &IR=1:1\rightarrow\qquad \omega_2\cong\omega_1\rightarrow\qquad \omega_2=\omega_1+\varepsilon^2\sigma_1 \\ &\Omega\cong\omega_1\rightarrow\qquad \Omega=\omega_1+\varepsilon^2\sigma_2 \end{aligned} \quad (6)$$

In case of symmetric resonator $\sigma_1=0$.

Finally, modulation equations are obtained as follows.

$$\acute{a}_1=\frac{1}{\omega_1}\left[-\omega_1\frac{1}{2Q}a_1+\alpha a_1a_2^2\sin(2\gamma_1)+F_0\sin\gamma_2\right] \quad (7)$$

$$\acute{\gamma}_2=\frac{1}{a_1\omega_1}\left[3\alpha a_1^3+\alpha a_1a_2^2[2+\cos(2\gamma_1)]+F_0\cos\gamma_2\right]-\sigma_2 \quad (8)$$

$$\acute{a}_2=\frac{1}{\omega_2}\left[-\omega_2\frac{1}{2Q}a_2-\alpha a_2a_1^2\sin(2\gamma_1)\right] \quad (9)$$

$$a_2\left(\acute{\gamma}_2-\acute{\gamma}_1+\sigma_2-\sigma_1-\frac{1}{\omega_2}\left[3\alpha a_2^2+\alpha a_1^2[2+\cos(2\gamma_1)]\right]\right)=0 \quad (10)$$

For the steady-state response

$$\begin{aligned} &\acute{a}_1=\acute{a}_2=\acute{\gamma}_1=\acute{\gamma}_2=0 \\ &\acute{\beta}_1=\sigma_2,\qquad \acute{\beta}_2=\sigma_2-\sigma_1 \end{aligned} \quad (11)$$

Above equations can be solved numerically.

Asymmetry in cross-section of a resonator can be modeled as an elliptical cross-section with radius ratio of $\frac{R_2}{R_1}$ Figure 1.

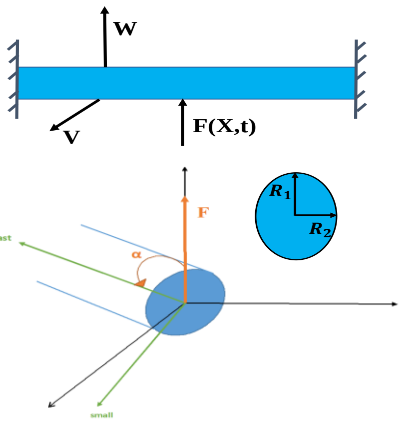

Figure 1. Asymmetric resonator with elliptical cross-section

Increasing or decreasing in radius ratio (asymmetric parameter) can lead to changes in internal resonance condition. As shown in figures 2-4, A small asymmetry in the cross-sectional area can lead to significant changes in the linear and non-linear vibration behavior of this type of nano resonators. By deliberately creating and controlling the asymmetry in the cross section, the internal resonance performance and therefore the vibration response of the resonator can be controlled for the desired applications. The results show that for asymmetries $\frac{R_2}{R_1} \leq 0.93$ and $\frac{R_2}{R_1} \geq 1.1$, the internal resonance performance is removed and destroyed.

Therefore, in cases where the internal resonance only results in the energy loss and low quality factor in the graphene resonators and carbon nanotubes, one can increase the asymmetry (for example by depositing the mass in the main inertia axes) and prevent the internal resonance and energy exchange between modes.

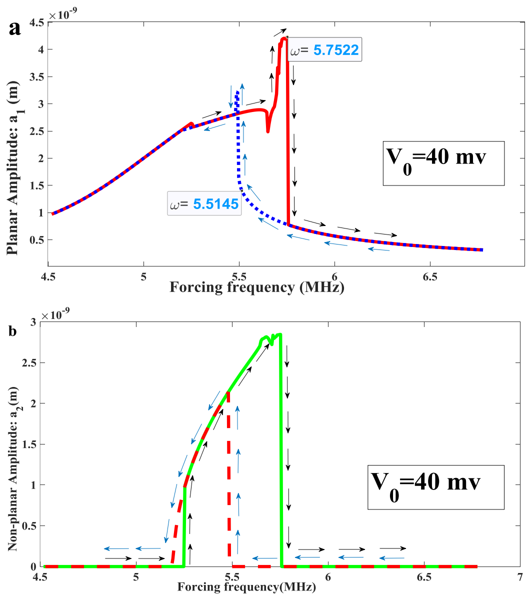

Figure 2. a. planar frequency response of ideal resonator in two dimensions b. Non-planar frequency response of ideal resonator in two dimensions

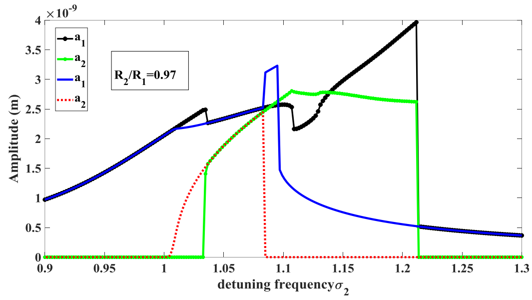

Figure 3. Non-planar frequency response in asymmetric resonator with $\frac{R_2}{R_1} = 0.97$

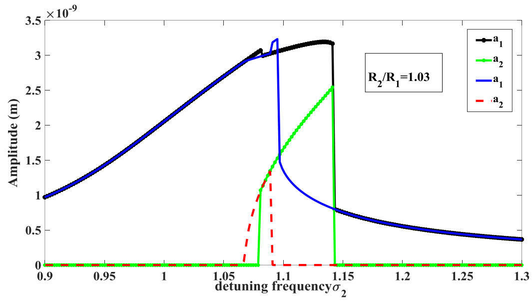

Figure 4. Non-planar frequency response in asymmetric resonator with $\frac{R_2}{R_1} = 1.03$

3. Conclusions

In this study, attempts were made to achieve a richer insight for designing and optimizing the performance of micro/nano scale resonators by examining the linear and non-linear behaviors of a nanoresonator under internal resonance conditions by using saddle-node and pitchfork bifurcations.

For example, internal resonance condition can be exploit to reduce the vibration amplitude in self-oscillating nanoresonators.

In addition, by intentionally creating asymmetry in the cross-section, the frequency region in which the internal resonance is activated can be changed according to the desired application.

The results show that the following cases can be considered for optimal design of nanoresonators:

- For enhancing the vibration amplitude of internal resonance, $a_2 \geq 2 \times 10^{-9}$, $0.96 \leq \frac{R_2}{R_1} \leq 1.06$
- For vanishing the vibration amplitude of internal resonance $\frac{R_2}{R_1} \leq 0.9$ and $\frac{R_2}{R_1} \geq 1.15$

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