Journal of Solid and Fluid Mechanics (JSFM), 13(4): 1-5, 2023

Shahrood University of Technology

Journal of Solid and Fluid Mechanics (JSFM)

DOI: 10.22044/JSFM.2023.12071.3628

Manufacturing Engineering of Iran

Effects of the rigid disk attached to the edges of the cylindrical shells on the natural frequencies of different modes

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Received: 08/16/2022 Revised: 07/03/2023 Accepted: 09/19/1402

Abstract
The vibration of cylindrical shells with rigid disks attached to the edges is investigated and the results are compared with those obtained under the common simplifying assumption that the edges are fixed at the interface of the rigid disk and the cylindrical shell. The shell is modeled using Sanders-Koiter shell theory, including the transverse shear deformation. The effect of the rigid disk on the edge displacements is also determined in a systematic manner using the kinematic relations of the rigid disk. To solve the problem, the semi-analytical finite element method is used and the stiffness and mass matrices of the element attached to the disks are determined completely for the first time. The reason why the disk affects the stiffness matrix is that some constraints appear between the displacement components of the shell edges due to the rigidity of the disk. Several numerical studies are conducted to investigate the influence of the mass properties of the rigid disks on different shell natural frequencies and mode shapes. Results show that the rigid disk can significantly change the natural frequencies of the modes with zero and one circumferential wave number.

Keywords: Cylindrical shell, rigid disk, semi-analytical finite element method, mode shapes.

1. Introduction

Cylindrical shells are one of the most common forms of structures used in many applications such as fuel containers, satellite and spacecraft, and launch vehicles. In most of these applications, the structure is under dynamic loads which require that the vibration characteristics of the shell is accurately determined during the design process.

The common task used for vibration analysis of the shell is isolating a single cylinder and fix the edges with some appropriate Boundary Conditions (BCs) such as the simply-supported or clamped BCs. However, in most cases, this approach does not accurately represent the actual constraints since the shell is connected to other bodies. Hence, to obtain the most accurate results, it is required that the whole structure, consisting of all the bays connected to each other is modeled. This, however, make the model computationally intensive and thus slows down the simulation process. An alternative approach that can compromise between the accuracy and the computational efficiency is to model adjacent bays as rigid bodies with specific inertial properties. This way, the constraining effects of the adjacent structure and also their mass effects would be accounted for without increasing the computational complexity.

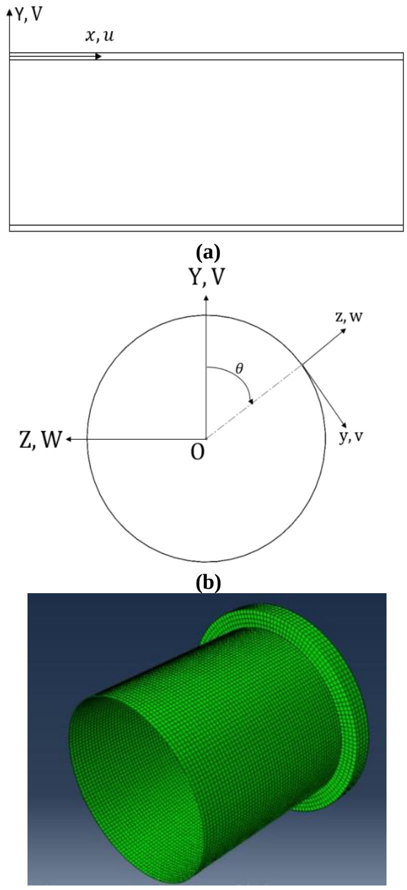
Effects of the various edge constraints have been considered in many studies. Zhou et al. [1] considered the effect of the elastic constraints on the vibration of cylindrical shells and Tang et al. [2] developed a model with more realistic BCs, by including the effect of the bolts at the ends of the cylindrical shell. A kind of multi-segment cylindrical shells is also considered by Qu et al. [3] and Tang et al. [4], where each segment had different thickness.

Regarding the effects of the attached rigid disk on the vibration of the cylindrical shell, the seminal work of Pellicano [5] can be mentioned, where both the theoretical and experimental studies are performed on the linear and nonlinear vibration of the cylindrical shell. The dynamic stability of the shell with the top mass was also considered by Pellicano and Avramov [6] and also by Pellicano [7], where again the experimental results were used to support their theoretical findings.

In all the above studies, the main focus was on the effect of the attached mass on the dynamics of the system and no numerical results are presented to indicate how much the common method of assuming fixed BCs at the connection with other bodies decreases the accuracy of the predicted frequencies, compared to the result obtained by accounting for the effect of inertial properties of connected bodies. Moreover, the methods of solution used in most of these studies were based on the Ritz-method with some appropriate approximating functions and the other powerful methods like the common hybrid finite-element method (FEM), used for the analysis of axisymmetric structures, has not been applied yet for the vibration analysis of cylindrical shells with the attached disk. Accordingly, in the present study, the free-vibration of cylindrical shell with attached masses at the ends is re-investigated by systematically deriving the governing equations of motion. For this purpose, the kinematic relations needed at the interface of the rigid disk and the cylinder are presented in-details, which would lead to some relations between the displacement components of the shell at the interface. These relations are the results of the rigidity of the disk that produces constraints between the shell's displacement components. Moreover, the effect of the disk inertial properties are also taken into account by considering the kinetic energy of the disk in the formulation. The Sanders-Koiter shell theory, with the transverse shear deformation is employed for modeling the shell and the solution is provided by the semi-analytical finite element method. The shear locking phenomenon is also avoided by using the field consistency approach. The main contribution of the present study lies in the presented numerical results and discussion, which are focused on the changes in the lowest natural frequencies and mode shapes of the cylindrical shells that occur when the rigid-disk is used at the end of the cylinder, compared to the simplifying assumption of fixed BCs at the edges connected to adjacent bodies. Moreover, the hybrid analytical-FEM method is used for the first time in the present study for vibration analysis of cylindrical shells with the attached disks. It will also be shown that the application of this method for the present problem is not completely straight forward and needs to apply some specific modifications to the stiffness and mass matrices.

2. Formulation

The geometry of the problem considered in the present study is shown in Fig. 1. According to this figure, the two coordinate systems, *xyz* and XYZ are used for describing the displacements of the shell and the disk, respectively. The *xyz* coordinate is the cylindrical coordinate with $y = R\theta$, and XYZ is the Cartesian coordinate, where its origin is located at the center of the disk in its interface with the cylinder. The displacement components of the disk are also denoted by U, V and W, which are displacements in *X*, *Y*, and *Z* directions. The displacements of the shell are also denoted by u, v, and w, which are given with respect to the *xyz* coordinate system.



(c)

Figure 1. The schematic of the cylindrical shell with the adopted coordinate systems (a) Side view (b) Front view (c) 3D view

The shell displacement field in the z-direction is then defined according to the first-order shear deformation theory as:

$$\begin{aligned} u &= u_0(x,\theta) + u_1(x,\theta)z \\ v &= v_0(x,\theta) + v_1(x,\theta)z \\ w &= w_0(x,\theta) \end{aligned} \tag{1}$$

Next the strain-displacement relationships based on the Sanders-Koiter shell theory [8], and the stress-strain relationships obtained for linear elastic isotropic materials in the plane-stress state [8] are used to derive the kinetic and strain energies of the shell. The kinetic energy of the disk would also be obtained as:

$$\begin{aligned} T_d = \frac{1}{2}m_d\dot{U}^2 + \frac{1}{2}m_d(\dot{V} + h_g\dot{\varphi})^2 \\ + \frac{1}{2}m_d(\dot{W} - h_g\dot{\psi})^2 \\ + \frac{1}{2}J_X\dot{\theta}^2 + \frac{1}{2}J_Y + \frac{1}{2}J_Z\dot{\varphi}^2 \end{aligned} \tag{2}$$

where , ψ, and φ are the infinitesimal Euler rotation angles about the X-, Y-, and Z-axis. Also J_X, J_Y, and J_Z

are mass moments of inertia of the disk and m_d is the mass of the disk.
In order to provide the solution, the displacement terms of the shell are taken as:

$$\begin{aligned} u_0 &= \tilde{U}_n(x)\cos(n\theta) \\ v_0 &= \tilde{V}_n(x)\sin(n\theta) \\ w_0 &= \tilde{W}_n(x)\cos(n\theta) \\ u_1 &= \tilde{\phi}_{un}(x)\cos(n\theta) \\ v_1 &= \tilde{\phi}_{vn}(x)\sin(n\theta) \\ n &= 0, 1, 2, \ldots \end{aligned} \tag{3}$$

where n is the circumferential wave number. Moreover, the functions of x in Eq. (1) are approximated using the Lagrangian shape functions corresponding to the three-node element [8]. To avoid the shear locking, according to the Field consistency approach, the function $\tilde{\phi}_{un}(x)$ that appear in the transverse shear strain would be approximated using lower-order shape functions [8, 9]. It is to be noted that since the three-node element is used in the FEM, for the number of N_e elements, there would be $2N_e + 1$ nodal values corresponding to each displacement terms in Eq. (3). Hence, considering all the five displacement terms, the total number of nodal values would be $10N_e$+5.
Next, the displacement compatibility conditions are imposed at the interface of the shell edge and the disk. This would yield the following relation between the displacement terms of the shell and the disk at the interface:

$$\begin{aligned} u_0 &= U - R\sin\theta\,\psi - R\cos\theta\,\varphi \\ v_0 &= V\sin\theta + W\cos\theta + \theta R \\ w_0 &= V\cos\theta - W\sin\theta \\ u_1 &= -\sin(\theta)\,\psi - \cos(\theta)\,\varphi \\ v_1 &= -\theta \end{aligned} \tag{4}$$

Substituting Eq. (3) into (4) and equating the coefficients of $\sin\theta, \cos\theta, \sin n\theta, \cos n\theta$ in both sides yield:

$$\begin{aligned} &\tilde{U}_0 = U, \tilde{V}_0 = -R\theta, \tilde{W}_0 = 0, \\ &\tilde{\phi}_{u0} = 0, \tilde{\phi}_{v0} = -\theta, \\ &\psi = \varphi = 0, V = W = 0 \end{aligned} \tag{5}$$

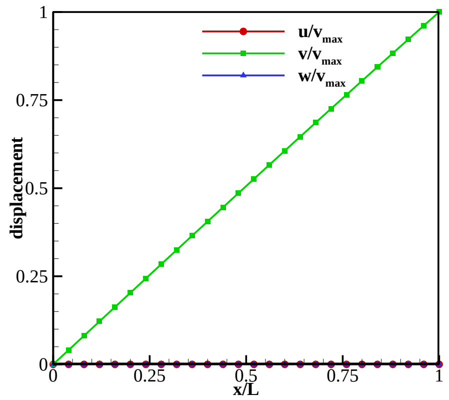
$$\begin{aligned} &\tilde{U}_1 = -\varphi R, \tilde{V}_1 = -V, \tilde{W}_1 = V, \quad \tilde{\phi}_{u1} = -\varphi, \\ &\tilde{\phi}_{v1} = 0 \\ &\psi = \theta = 0, W = U = 0, \end{aligned} \tag{6}$$

$$\begin{aligned} &\tilde{U}_n = 0, \tilde{V}_n = 0, \quad \tilde{W}_n = 0, \tilde{\phi}_{un} = 0, \quad \tilde{\phi}_{vn} = \\ &0\ (n > 1), \\ &\theta = \psi = \varphi = 0, U = V = W = 0 \end{aligned} \tag{7}$$

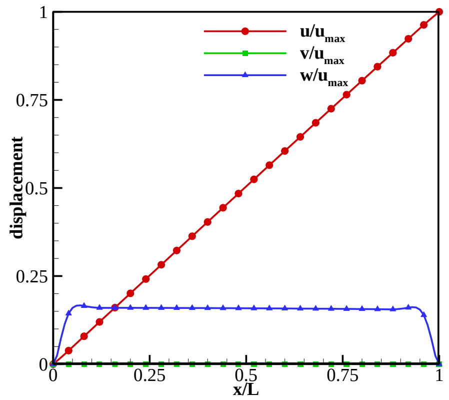
where Eq. (5), (6) and (7) would affect the BCs for the modes with $n = 0$, $n = 1$, $n > 1$ respectively. These BCs are then applied to the mass and stiffness matrices, changing some of the columns and rows corresponding to the element attached to the disk.

3. Numerical results and discussions

First, the model developed in the present study is verified by comparing with the result of the Abaqus for some natural frequencies and mode shapes. Then the mode shapes of the cylindrical shell with the attached mass on the right edge and the clamped left edge are plotted in Fig. 2 for some lower-frequency modes. Fig. 2 (a) and (b) shows that the circumferential and axial displacements are dominant respectively in the first and second modes, corresponding to $n = 0$, which implies that these modes are in fact the torsional and axial modes. For the first mode corresponding to $n = 1$ (Fig. 2(c)) the radial displacement is dominant, meaning that this mode is a beam-like mode of the shell, where the cylinder experiences both the bending and torsional deformations. In the second mode with $n = 1$ (Fig. 2(d)), the motion is again dominated by the axial deformation, with lower participation of the bending and torsional motions.



(a)



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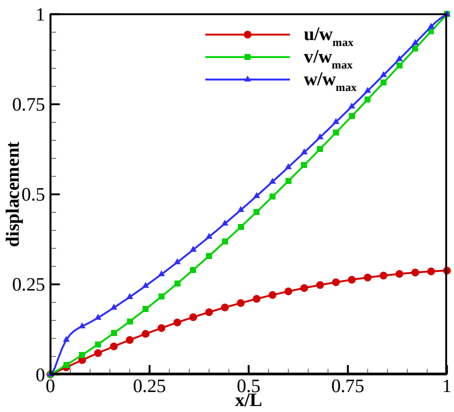

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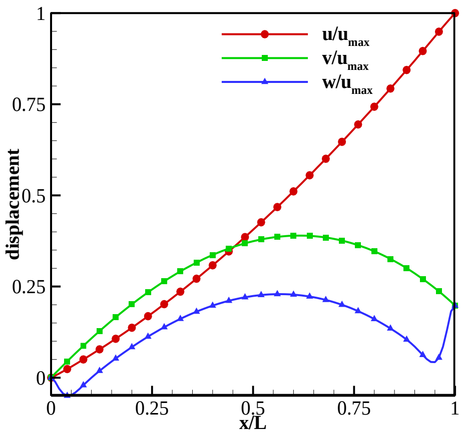

(d)

Figure 2. The first two mode shapes of the shell with the disk attached to right edge (a) First mode with $n = 0$ (b) Second mode with $n = 0$, (c) First mode for $n = 1$, (d) Second mode for $n = 1$

Next, a numerical study is performed in Fig. 3 on the variation of the two lowest frequencies corresponding to different values of n, for a cylinder with an attached mass to its right edge. The result obtained for the shell with both-ends clamped is also included in this figure. The figure shows that how the mass attached to edge can change the lowest fundamental frequency of the cylinder from the mode with $n = 8$ to the mode with $n = 1$ or $n = 0$.

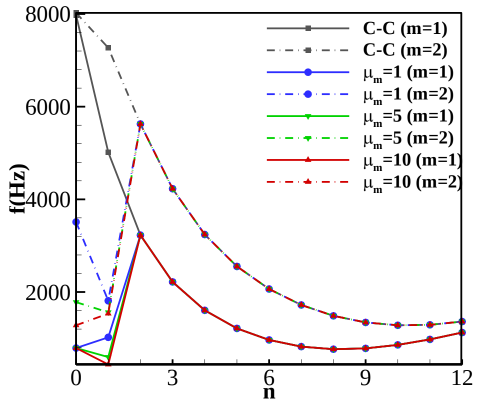

Figure 3. Variation of the first two frequencies with n obtained for three different disk masses

4. Conclusions

The hybrid analytical FEM is used to solve the free-vibration problem of a cylindrical shell with rigid disks attached to the edges. Results showed that the lowest fundamental frequency of the shell can be drastically changed from a mode with higher circumferential wave number to a mode with one circumferential wave number (beam-like mode), when the inertial properties of the adjacent bodies are considered in the analysis of a cylindrical shell. Hence, using clamped BCs at the edges where the shell is attached to other parts can lead to unrealistic results. The mode shapes corresponding to the lowest frequencies are also considerably affected by the attached disk.

5. References

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