Journal of Solid and Fluid Mechanics (JSFM), 13(4): 1-4, 2023

Shahrood University of Technology

Journal of Solid and Fluid Mechanics (JSFM)

DOI: 10.22044/JSFM.2023.13315.3758

Manufacturing Engineering of Iran

Nonlinear Thermo-Elastic Analysis of FG Cylinder with Temperature-Dependent Properties with Perturbation and Differential Quadrature Methods

Mahdi Dehghan[1], Behrouz Shahriari[2,*], Amin Moosaei[3]

[1]Ph.D., Faculty of Mechanics, Malek Ashtar University of Technology, Isfahan, Iran.
[2]Assist. Prof., Faculty of Mechanics, Malek Ashtar University of Technology, Isfahan, Iran.
[3]Assoc. Prof., Department of Mechanical Engineering, Yasouj University, Yasouj, Iran.

*Corresponding author: shahriari@mut-es.ac.ir

Received: 06/30/2023 Revised: 08/11/2023 Accepted: 10/01/2023

Abstract

In this article, thermo-elastic analysis of a thick hollow cylinder under internal pressure is investigated. The material properties are considered to be temperature-dependent and functionally graded in the radial direction. This temperature dependency causes the heat conduction equations to be nonlinear. The perturbation theory is performed for solving nonlinear heat conduction equations of the cylinder. Arbitrary combined thermal boundary conditions containing temperature and temperature gradient are considered. As the temperature field is extracted, it can be used to solve decoupling thermal stress governing equations. These thermo-elastic equations and related boundary conditions are discretized in strong form using differential quadrature method. Numerical results extracted from the suggested semi-analytical approach in a linear regime are then compared to the existing results. It is depicted that the presented method benefits from the low computational and high convergence behavior. The stress distribution of the cylinder is computed by considering the approximation of the temperature in zeroth-order, one-order as well as second-order. In order to achieve proper accuracy, more terms of the perturbation approximation series should be considered for the higher temperature difference between the inner and outer surfaces of the cylinder.

Keywords: Perturbation theory; Differential quadrature (DQ) method; FG cylinder; Temperature-dependent material properties; Thermo-elastic analysis.

1. Introduction

Functionally graded materials (FGMs) are new multi-functional materials increasingly used in many fields of engineering especially in high temperature applications such as thermo- mechanical loading structures, aircraft, spacecraft and other engineering structures in high temperature environments. The important advantages offered by functionally graded materials over conventional composite materials are eliminated the interface problems of conventional composite materials and the stress distribution becomes mitigated. Since FGM's are often to be used under thermal loadings, the thermoelastic analysis of structures made of FGM's is of paramount importance.

Physical reality for both homogeneous and functionally graded materials is the dependence of material properties on temperature. This is more pronounced especially when large temperature differences are involved in the problem. That is, the dependence of physical properties on temperature become negligible at lower temperature differences. Nevertheless, there are certain applications in which one cannot neglect this fact and has to take temperature-dependent material properties into account. However, this makes the analysis very complicated as the governing differential equations become nonlinear [1-3]. One example of a particularly important application of the thick wall hollow cylinder to nuclear reactor pressure vessel. The nuclear reactor pressure vessel is an important component of a nuclear power plant. It has been used in harsh environments such as high temperature as well as the high pressure [4]. Moosaie [3] investigated the nonlinear thermos-mechanical responses of an incompressible FG hollow cylinder with temperature-dependent material properties using perturbation technique. Thermal stress analysis of FG hollow cylinder due to radially symmetric loads accomplished by Jabbari et al. [5].

In this paper, the nonlinear heat conduction equation of a thick hollow cylinder is analyzed using the perturbation method, specifically Poincare's technique. Subsequently, the GDQ method is employed to extract the thermal stresses of the FG cylinder under thermo-mechanical loadings.

2. Governing Equations of the Problem

2.1. Nonlinear Heat Conduction Equation

In this section, the differential equation governing the heat conduction problem in a hollow cylinder made of FGM with temperature-dependent material properties along with the required boundary conditions are presented. Material properties of the cylinder are assumed to be function of the temperature field and vary in a functionally graded manner in the thickness direction as the followings:

$$E = E(\vartheta, r) = (E_0 - E_1\vartheta - E_2\vartheta^2)\left(\frac{r}{R_o}\right)^{m_1} \tag{1}$$

$$\alpha = \alpha(\vartheta, r) = (\alpha_0 + \alpha_1\vartheta)\left(\frac{r}{R_o}\right)^{m_2} \tag{2}$$

$$\lambda_t = \lambda_t(\vartheta, r) = (\lambda_{t0} - \lambda_{t1}\vartheta)\left(\frac{r}{R_o}\right)^{m_3} \tag{3}$$

Whereϑ,E,α, and λ_t introduce temperature field, young's modulus, coefficient of thermal expansion, and heat conduction coefficient, respectively. Nonlinear equation of the heat conduction for FG cylinder can be written as bellow

$$\left[(1+m_3)(1-\varepsilon\vartheta) - \varepsilon r\frac{d\vartheta}{dr}\right]\frac{d\vartheta}{dr} + r(1-\varepsilon\vartheta)\frac{d^2\vartheta}{dr^2} = 0 \tag{4}$$

In which, $\varepsilon = \frac{\lambda_{t1}}{\lambda_{t0}} \ll 1$. For related boundary conditions in general form can be referred to

$$\begin{cases} H_{11}\vartheta(r = r_i) + H_{12}\vartheta'(r = r_i) = f_1 \\ H_{21}\vartheta(r = r_o) + H_{22}\vartheta'(r = r_o) = f_2 \end{cases} \tag{5}$$

Now, the perturbation method can be used for solving above nonlinear equation. So, for temperature variable we have

$$\vartheta = \hat{\vartheta}_0 + \varepsilon\hat{\vartheta}_1 + \varepsilon^2\hat{\vartheta}_2 + \ldots = \sum_{k=0}^{\infty}\varepsilon^k\hat{\vartheta}_k \tag{6}$$

Different order of the above expansion with respect to the ε parameter can be truncated to approximate the temperature field variable.

$$\vartheta_0 = \hat{\vartheta}_0 \tag{7}$$

$$\vartheta_1 = \hat{\vartheta}_0 + \varepsilon\hat{\vartheta}_1 \tag{8}$$

$$\vartheta_2 = \hat{\vartheta}_0 + \varepsilon\hat{\vartheta}_1 + \varepsilon^2\hat{\vartheta}_2 \tag{9}$$

Substituting (6) into (4) and grouping all terms with the same power of ε we get

$$\begin{aligned} &\varepsilon^0\left[r\frac{d^2\hat{\vartheta}_0}{dr^2} + (1+m_3)\frac{d\hat{\vartheta}_0}{dr}\right] \\ &+\varepsilon^1\Bigg[r\frac{d^2\hat{\vartheta}_1}{dr^2} + (1+m_3)\frac{d\hat{\vartheta}_1}{dr} \\ &-\hat{\vartheta}_0\left(r\frac{d^2\hat{\vartheta}_0}{dr^2} + (1+m_3)\frac{d\hat{\vartheta}_0}{dr}\right) - r\frac{d\hat{\vartheta}_0}{dr}\frac{d\hat{\vartheta}_0}{dr}\Bigg] \\ &+\varepsilon^2\Bigg[r\frac{d^2\hat{\vartheta}_2}{dr^2} + (1+m_3)\frac{d\hat{\vartheta}_2}{dr} \\ &-\hat{\vartheta}_0\left(r\frac{d^2\hat{\vartheta}_1}{dr^2} + (1+m_3)\frac{d\hat{\vartheta}_1}{dr}\right) \\ &-\hat{\vartheta}_1\left(r\frac{d^2\hat{\vartheta}_0}{dr^2} + (1+m_3)\frac{d\hat{\vartheta}_0}{dr}\right) - 2r\frac{d\hat{\vartheta}_0}{dr}\frac{d\hat{\vartheta}_1}{dr}\Bigg] \\ &+O(\varepsilon^3) = 0 \end{aligned} \tag{10}$$

Since above relation is to be satisfied for arbitrary value of ε parameter, bracketed terms are to vanish. We start the solution procedure with reduced equation$O(\varepsilon^0)$.

$$r\frac{d^2\hat{\vartheta}_0}{dr^2} + (1+m_3)\frac{d\hat{\vartheta}_0}{dr} = 0 \tag{11}$$

This is a Cauchy-Euler differential equation whose general solution reads

$$\hat{\vartheta}_0(r) = C_1^0 + \frac{C_2^0}{r^{m_3}} \tag{12}$$

$O(\varepsilon^1)$ term of (10) executed by vanishing the multiplier of ε^1 and reaches to

$$\begin{aligned} &r\frac{d^2\hat{\vartheta}_1}{dr^2} + (1+m_3)\frac{d\hat{\vartheta}_1}{dr} = \\ &\hat{\vartheta}_0\left(r\frac{d^2\hat{\vartheta}_0}{dr^2} + (1+m_3)\frac{d\hat{\vartheta}_0}{dr}\right) \\ &\qquad + r\frac{d\hat{\vartheta}_0}{dr}\frac{d\hat{\vartheta}_0}{dr} \end{aligned} \tag{13}$$

This is a nonhomogeneous Cauchy-Euler differential equation whose general solution reads

$$\hat{\vartheta}_1(r) = C_1^1 + \frac{C_2^1}{r^{m_3}} + \frac{(C_2^0)^2}{2r^{2m_3}} \tag{14}$$

Finally, in (10) by equating the multiplier of the ε^2 to zero the remaining equation of $O(\varepsilon^2)$ obtains

$$\begin{aligned} &r\frac{d^2\hat{\vartheta}_2}{dr^2} + (1+m_3)\frac{d\hat{\vartheta}_2}{dr} = \\ &\hat{\vartheta}_0\left(r\frac{d^2\hat{\vartheta}_1}{dr^2} + (1+m_3)\frac{d\hat{\vartheta}_1}{dr}\right) \\ &+\hat{\vartheta}_1\left(r\frac{d^2\hat{\vartheta}_0}{dr^2} + (1+m_3)\frac{d\hat{\vartheta}_0}{dr}\right) \\ &\qquad + 2r\frac{d\hat{\vartheta}_0}{dr}\frac{d\hat{\vartheta}_1}{dr} \end{aligned} \tag{15}$$

This is also a nonhomogeneous Cauchy-Euler differential equation whose general solution reads

$$\hat{\vartheta}_2(r) = C_1^2 + \frac{C_2^2}{r^{m_3}} + \frac{\gamma}{2r^{2m_3}} + \frac{(C_2^0)^3}{2r^{3m_3}} \tag{16}$$

where

$$\gamma = C_2^0(C_1^0 C_2^0 + 2C_2^1) \tag{15}$$

2.2. Thermo-elastic Analysis Using GDQ Method

Thermal stresses along the thickness of the cylinder are assumed to be produced due to temperature gradient and are represented by the plane-strain theory. Young's modulus as well as the coefficient of thermal expansion vary gradually with respect to the radial spatial variable. These properties are considered to be functions of the temperature variable. Whereas, the poisson's ratio is assumed to be constant. Axisymmetric governing equations of thermal stresses of a cylinder can be written as

$$\frac{d\sigma_{rr}}{dr} + \frac{1}{r}(\sigma_{rr} - \sigma_{\theta\theta}) = 0 \tag{16}$$

Kinematic equations as a relationship between strains and displacement components are to be

$$\varepsilon_{rr} = \frac{du}{dr}, \quad \varepsilon_{\theta\theta} = \frac{u}{r} \tag{17}$$

Constitutive relations by considering the plane-strain theory become

$$\begin{aligned} \sigma_{rr} &= (\lambda + 2\mu)\varepsilon_{rr} + \lambda\varepsilon_{\theta\theta} - (3\lambda + 2\mu)\alpha\vartheta(r) \\ \sigma_{\theta\theta} &= (\lambda + 2\mu)\varepsilon_{\theta\theta} + \lambda\varepsilon_{rr} - (3\lambda + 2\mu)\alpha\vartheta(r) \end{aligned} \tag{18}$$

In which

$$\lambda = \frac{\nu E(\vartheta, r)}{(1+\nu)(1-2\nu)}, \quad \mu = \frac{E(\vartheta, r)}{2(1+\nu)} \tag{19}$$

Substituting (1), (2), (17)-(19) into (16) the Lame-navier thermo-elastic governing equations of FG hollow thick cylinder with temperature-dependent material properties yields

$$\begin{aligned} &(\lambda + 2\mu)\frac{d^2u}{dr^2} + \left(\frac{d\lambda}{dr} + 2\frac{d\mu}{dr} + \frac{\lambda}{r} + \frac{2\mu}{r}\right)\frac{du}{dr} + \left(\frac{1}{r}\frac{d\lambda}{dr} - \frac{1}{r^2}\lambda - \frac{2\mu}{r^2}\right)u \\ &= \left[\left(3\frac{d\lambda}{dr} + 2\frac{d\mu}{dr}\right)\alpha + (3\lambda + 2\mu)\frac{d\alpha}{dr}\right]\vartheta + (3\lambda + 2\mu)\frac{d\vartheta}{dr} \end{aligned} \tag{20}$$

In this paper, a differential quadrature method (DQM) is used to discretize the thermo-elastic governing equations of FG cylinder (Figure 1).

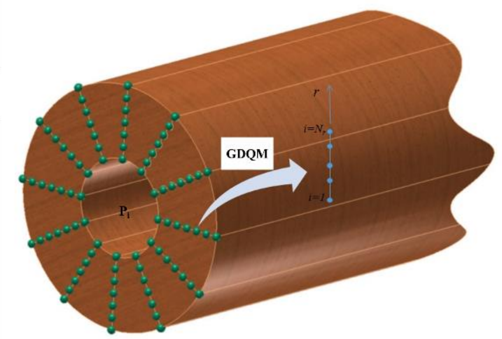

Figure 1. Discretization of the domain

3. Results and Discussion

In this section, some selected results obtained by proposed semi-analytical method are presented considering the following material constants

$$\begin{aligned} &E_0 = 2.1 \times 10^5 \text{ MPa}, \quad E_1 = 27.5 \text{ MPa/°C}, \\ &E_2 = 0.141 \text{ MPa/°C}^2, \\ &\alpha_0 = 1.2 \times 10^{-5} \text{ °C}^{-1}, \quad \alpha_1 = 1 \times 10^{-8} \text{ °C}^{-2}, \\ &\lambda_{t0} = 50.16 \text{ W/m°C}, \lambda_{t1} = 0.0293 \text{ W/m°C}^2 \end{aligned} \tag{21}$$

For better demonstration, the obtained results are compared to those presented in Ref [5] and depicted in Figure 2.

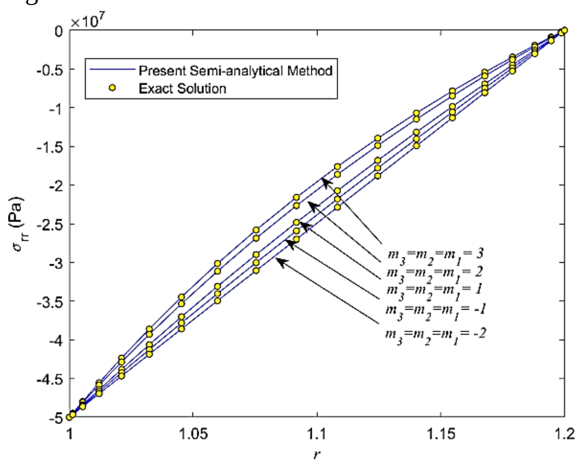

Figure 2. Distribution of the radial stress along the cylinder thickness

Thereby, distribution of the temperature field of the FG cylinder in terms of various orders of the approximation parameter ε, from zeroth order up to second order is presented in Figure 3. As the approximation order increases, the temperature distribution converges. The following boundary conditions are considered for this purpose.

$$\begin{aligned} &\vartheta(r = R_i) = 0, \quad \vartheta(r = R_o) = 1000 \text{ °C} \\ &\sigma_{rr}(r = R_i) = 0, \quad \sigma_{rr}(r = R_o) = 0 \end{aligned} \tag{22}$$

Distribution of the radial stress along the cylinder thickness by considering three approximation order is presented in Figure 4.

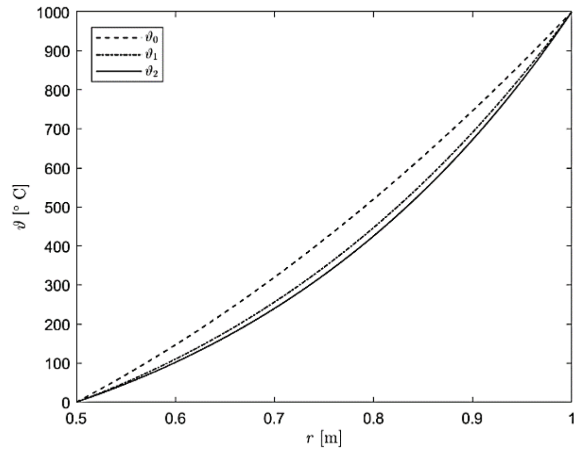

Figure 3. Distribution of the temperature field for various approximations with $\vartheta_o = 1000$ ℃ ،$\vartheta_i = 0$ ℃ $(m_3 = -2)$

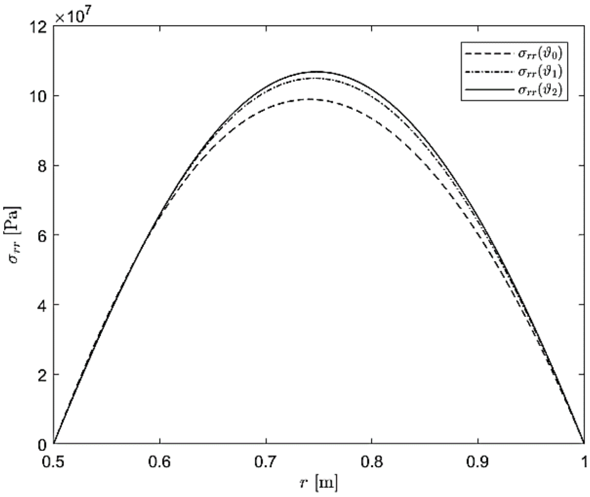

Figure 4. Radial stress for different temperature approximations, $(m = 3)$.

4. Conclusions

In this paper, a suitable framework has been provided based on the perturbation method and differential quadrature technique for nonlinear thermoelastic analysis of an FG cylinder under internal pressure. Material properties have been considered to be functionally graded in radial direction and temperature-dependent. This dependency make the governing heat conduction equations of the cylinder to be nonlinear. For solving the nonlinear governing equations, the perturbation theory and asymptotic methods are used. The temperature field can be approximated with respect to the various orders of the perturbation parameter. Desired thermoelastic governing equations are successfully discretized by using the DQ method. Afterwards, the distribution of the temperature field and thermal stress along the thickness direction is presented in different approximation order. In the case of the high-temperature difference between the inner and outer surface of the cylinder, it was shown that to reach more precision we need to use higher-order approximation terms. The nonlinear temperature variable has the most influence on Young's modulus and the least influence on the coefficient of thermal expansion, respectively.

5. References

[1] Arefi M (2013) Nonlinear thermoelastic analysis of thick-walled functionally graded piezoelectric cylinder. *Acta Mech.* 224 (11): 2771-2783.

[2] Arefi M, Faegh R K, Loghman A (2016) The effect of axially variable thermal and mechanical loads on the 2D thermoelastic response of FG cylindrical shell. *J. Therm. Stresses* 39 (12): 1539-1559.

[3] Moosaie A (2016) A nonlinear analysis of thermal stresses in an incompressible functionally graded hollow cylinder with temperature-dependent material properties. *Eur. J. Mech. A/Solids* 55: 212-220.

[4] Zhou L, Dai J, Li Y, Dai X, Xie C, Li L, Chen L (2022) Research Progress of Steels for Nuclear Reactor Pressure Vessels. *Materials* 15 (24): 8761.

[5] Jabbari M, Sohrabpour S, Eslami M (2002) Mechanical and thermal stresses in a functionally graded hollow cylinder due to radially symmetric loads. *Int. J. Press. Vessels Pip.* 79 (7): 493-497.