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Error estimation based on stress recovery by adaptivity in nonlinear problems of elasto-plasticity behavior by isogeometric method

Ali Shahini[1], Ahmad Ganjali[2], Aboozar Mirzakhani[2]

[1]Ph.D. Student, Department Of Civil Engineering, Shahrood Branch, Islamic Azad University, Shahrood, Iran.
[2]Assist. Prof, Department Of Civil Engineering, Shahrood Branch, Islamic Azad University, Shahrood, Iran.

*Corresponding author: Ahmad.ganjali@iau-shahrood.ac.ir

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Abstract
In this research, the efficiency of error estimation based on two methods of recovering the equilibrium stress of patches and superconvergent points in guiding the adaptive solution of nonlinear problems by isogeometric method has been investigated. Also, by analyzing elasto-plastic problems, and adaptive solution by temperature gradient method, the stress improvement process has been investigated. The method of the adaptive solution of this research is based on the movement of control points and it is used in stress recovery, taking into account the difference between the exact stress level and the stress level obtained from the isogeometric analysis for each element, as a criterion to determine the amount of error in it. The element is obtained. For this purpose, the modeling of two problems in the non-linear range, which has an exact solution, has been considered. The results have shown that the total norm difference of exact and approximate error in both stress recovery methods used is more than 33% and in the direction of improving the network of control points. Also, the method by equilibrium patches is more effective than the method based on superconvergent points, which can be used as a suitable solution to improve the stress field.

Keywords: Equilibrium Patches, Superconvergent Points, Error Estimation, Isogeometric Analysis, Improving the Network of Control Points.

1. Introduction

Today, with significant advances in the science of geometry modeling with the help of computers, creating the geometry of engineering problems has become more accurate, and one of the most important of these methods is the isogeometric method, which is based on the use of the development of NURBS[1] functions [1]. In this century, using the maximum capacity of materials and creating economical and at the same time safe designs has led to the expansion of analytical methods in the field of non-linear behavior of structures. In this regard, many researchers have modeled numerical algorithms such as Newton-Raphson, which was founded by Turner et al. And in the continuation of this research, researchers such as Zienkiewicz [2] developed non-linear methods in the finite element method.

Error is always considered an inseparable part of numerical analysis and has always caused researchers to worry about the reliability of the results. Therefore, in this research, the solution of non-linear problems based on error evaluation and the norm criteria of energy error has been investigated. And to achieve a more accurate solution to the problems, the adaptive solution has been used with the help of the displacement model of control points developed by Mirzakhani et al. [3].

In this research, the effectiveness of error estimation methods based on stress recovery [7-4] in the analysis of non-linear problems by the isogeometric method has been investigated. Also, based on the concepts of equilibrium of patches (RNIEP[2]) and concepts of superconvergent points (RNISP[3]), the refinement effect based on the displacement of control points based on thermal gradient (t-r-refinement[4]) has been improved and the development of mathematical relations of problems in the plane stress state and plane strain has been done. In the continuation of the results of the analysis, it is compared with its classical solution coded in the Fortran program environment, and its efficiency is examined in two sample problems.

[1] Non-Uniform Rational B-splines (NURBS).
[2] Recovery Stress in Nonlinear Isogeometric by Equilibrium Patches Analysis.
[3] Recovery Stress in Nonlinear Isogeometric by Superconvergent Points Analysis.
[4] Thermal Gradient based Remesh Refinement.

2. Extracting the relationships and equations

2.1 Nonlinear isogeometric analysis, concept, and formulation

The hardness of each patch of the element in each repetition is calculated using the Gauss integration method in the normal coordinate system as equation (1) (to clarify the development process of the above relations, reading reference [9] is recommended).

$$\mathbf{K}_{T(patch)} = \sum_{j=1}^{m}\sum_{i=1}^{n}\mathbf{B}^T(r,s)\mathbf{D}_{ep}\mathbf{B}(r,s)\,detJ_1 detJ_2 w_i w_j \qquad (1)$$

2.2 RNISP method in nonlinear isogeometric analysis

In general, in this method, the improved stress field for each stress component in each region is considered as a hypothetical level. This hypothetical level has been obtained by using NURBS shape functions that are used to estimate the unknown function (displacement). According to the functions of the shape of NURBS, this surface can be expressed in the form of equation (2) within each region [4]:

$$\boldsymbol{\sigma}^* = \sum_{i=1}^{m}\sum_{j=1}^{n}\mathbf{R}_{i,j}(u,v).\mathbf{P}_{i,j}$$

$$\mathbf{F}(\mathbf{P}) = \sum_{j=1}^{k_y}\sum_{i=1}^{k_x}(\boldsymbol{\sigma}^*_{i,j} - \bar{\boldsymbol{\sigma}}_{i,j})^2$$

$$\frac{\partial \mathbf{F}(\mathbf{P})}{\partial \mathbf{P}_{i,j}} = 0 \quad \Rightarrow \mathbf{AP} = \mathbf{B} \quad \Rightarrow \mathbf{P} = \mathbf{A}^{-1}\mathbf{B} \qquad (2)$$

$$\mathbf{A} = \sum_{i=1}^{K}\mathbf{R}_i\mathbf{R}_i^T \quad ; \quad \mathbf{B} = \sum_{i=1}^{K}\mathbf{R}_i\bar{\boldsymbol{\sigma}}_i$$

By having the coordinates of the control points, the corresponding stress level is also obtained.

2.3 RNIEP method in nonlinear isogeometric analysis

In this method, similar to the methods presented in [5] and [6] to improve the finite element stress field, by equivalently placing the forces acting on a patch of the computational space obtained from isogeometric analysis and the recovered stress field. an improved stress level can be obtained for each stress component. According to equation (3) by determining the vector of control points ($\mathbf{P}^*_\alpha$) for each stress component, the corresponding stress level is also obtained, and this recovered stress level is more accurate than the isogeometric stress level.

$$\mathbf{P}^*_\alpha = \left[\mathbf{C}_\alpha{}^T\mathbf{C}_\alpha + \sum_{e=1}^{Nel}\left(\mathbf{C}_\alpha\mathbf{C}_\alpha\right)_e\right]^{-1} \times \left[\mathbf{C}_\alpha{}^T\mathbf{F}^{iso}_\alpha + \sum_{e=1}^{Nel}\left(\mathbf{C}_\alpha{}^T\mathbf{F}^{iso}_\alpha\right)_e\right] \qquad (3)$$

2.4 Correlation of the theory of behavior of deformable elastoplastic materials

In this research, the relationship between the theory of the behavior of deformable elastoplastic materials, using Hooke's law and the von Mises yield criterion, is considered. Also, in order to end the iteration steps, the force convergence criterion of equation (4) [8] has been used.

$$\mathbf{R} = \frac{\left\|\left[\sum_{i=1}^{n_{dof}}\left(\boldsymbol{\Psi}^r{}_i\right)^2\right]^{1/2}\right\|}{\left\|\left[\sum_{i=1}^{n_{dof}}(\mathbf{F}_i)^2\right]^{1/2}\right\|} < TOLER \qquad (4)$$

2.5 Energy error norm in the nonlinear isogeometric method

The energy error norm has been calculated using the Gauss integration method for each element in a non-linear mode and at the end of each step of load increase in the form of equation (5), [9]:

$$\|e^*\| = \left[\sum_{j=1}^{m}\sum_{i=1}^{n}\frac{(\boldsymbol{\sigma}^* - \bar{\boldsymbol{\sigma}})^T\mathbf{D}_{ep}{}^{-1}(\boldsymbol{\sigma}^* - \bar{\boldsymbol{\sigma}}).}{|J_1|.|J_2|\,w_i.w_j}\right]^{\frac{1}{2}} \qquad (5)$$

2.6 Adaptivity solution by using a thermal gradient (t-r-refinement)

According to equations (6), in this method, energy error norm is calculated using isogeometric analysis, and the contribution of each control point from its vicinity is determined using Voronoi diagram. Then, the displacement of the control points is assumed by assuming a truss structure under a thermal gradient proportional to the errors of each control point. The analysis of this structure under temperature changes creates a new arrangement of control points, which in subsequent iterations of isogeometric and thermoelastic calculations ultimately leads to a better distribution of errors in the domain of problems and an optimal displacement of control points [3].

$$T_i = \left\{x \in \Re : d(X, X_i) < d(X, X_j)\,\forall j \neq i\right\}, \qquad (6)$$
$$\delta = L.\alpha.\Delta T, \ \delta = \frac{FL}{AE}$$

Figure 1 shows the adaptive solution algorithm for nonlinear elastoplastic problems based on the equilibrium of patches and superconvergent points using isogeometric method.

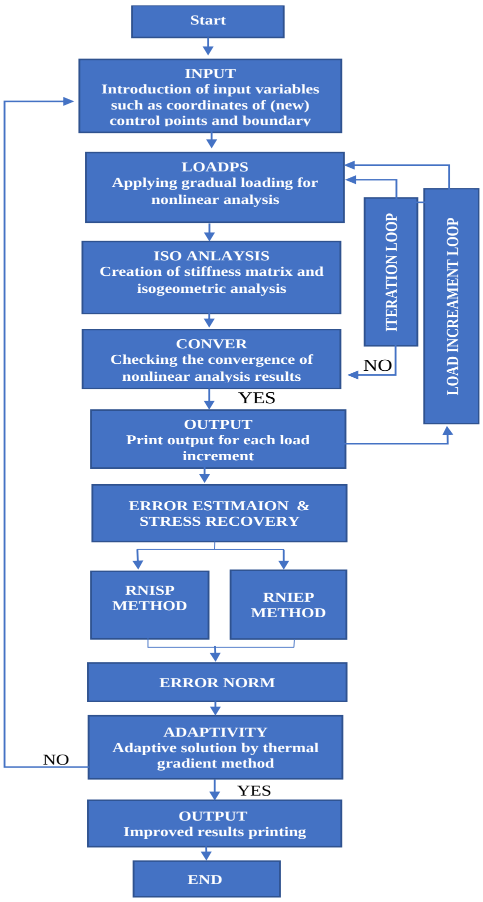

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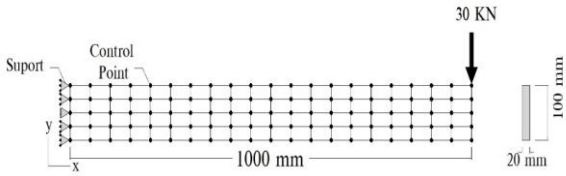
Figure 1. Adaptive solution algorithm of elastoplastic nonlinear problems based on the equilibrium of patches and superconvergent points by isogeometric method.

3. Discussion and Results

In this section, two sample problems have been presented to demonstrate the effectiveness of the t-r-refinement in guiding stress recovery methods using equilibrium by patches and superconvergent points based on nonlinear isogeometric analysis. Also, to further show the effect of t-r-refinement, first-degree shape functions are used and their results in stress improvement and error estimation are compared.

3.1 The first issue is the cantilever beam under the influence of concentrated load.

Figure 2-a the geometric characteristics of the cantilever beam are shown as a first problem header and Figure 2-b shows the comparison and matching of the accurate and improved σ_{XX} stress in the cantilever beam using the RNISP and RNIEP methods, using the definition of moving the control points, and it indicates that the process of stress distribution and using the definition to the method of moving control points (t-r-refinement) is improving after two cycles of refinement. And compared to the isogeometric solution (solution without stress recovery and adaptive solution), it has shown a more accurate solution.



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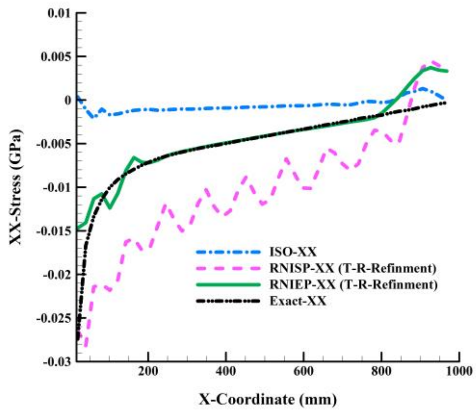

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Figure 2. a) The geometrical and boundary conditions of a cantilever beam and b) comparison of σ_{xx} stress matching, exact and improved with RNIEP and RNISP methods in cantilever beam.

Table 3, shows the total difference between exact and approximate error norms after two cycles of t-r-refinement. And as it has been observed, after two cycles of t-r-refinement, the difference between exact and approximate error norm has decreased in the cantilever beam. Also, the amount of this error in the RNIEP method is less than in the RNISP method, and it is a sign of the accuracy of the error calculator using the RNIEP method in identifying and calculating the error, and the RNIEP error estimator has correctly guided the control points network algorithm.

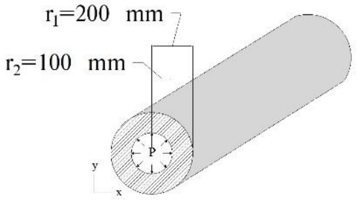
Table 1. The sum of the exact and approximate error norm difference in the cantilever beam after one and two cycles of improving the network of control points using the RNISP and RNIEP methods.

The sum of the difference between exact and approximate error norms			
Improving the network of control points (after the first cycle of refinement)		Improving the network of control points (after the second cycles of refinement)	
RNIEP	RNISP	RNIEP	RNISP
0.002	0.003	0.30	0.36

3.2 Second problem Elasto-Plastic analysis of thick-walled cylindrical due to internal pressure.

Figure 3-a the geometric characteristics of the thick-

walled cylinder are shown as a first problem header and Figure 3-b shows the comparison and matching of the accurate and improved σ_r stress in the thick-walled cylinder using the RNISP and RNIEP methods, using the definition of moving the control points, and it indicates that the process of stress distribution and using the definition to the method of moving control points (t-r-refinement) is improving after three cycles of refinement. And compared to the isogeometric solution (solution without stress recovery and adaptive solution), it has shown a more accurate solution.



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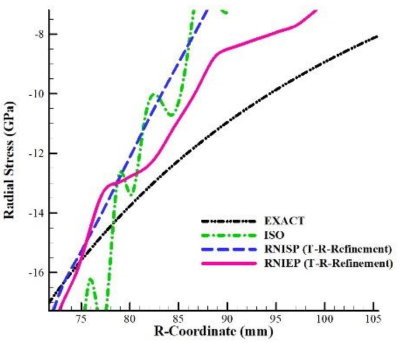

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Figure 3. a) The geometrical and boundary conditions of a thick-walled cylinder and b) comparison of σ_r stress matching, exact and improved with RNIEP and RNISP methods in thick-walled cylinder.

Table 2. The sum of the exact and approximate error norm difference in the thick-walled cylinder after one and two cycles of improving the network of control points using the RNISP and RNIEP methods.

The sum of the difference between exact and approximate error norms					
Improving the network of control points (after the first cycle of refinement)		Improving the network of control points (after the second cycles of refinement)		Improving the network of control points (after the third cycles of refinement)	
RNISP	RNIEP	RNISP	RNIEP	RNISP	RNIEP
2.73	1.47	1.41	0.32	1.08	0.22

4. Conclusions

In this research, with the approach of using isogeometric method in solving non-linear problems, as well as using stress recovery based on the equilibrium of patches and superconvergent points, the error estimation was discussed and the effect of the error estimators used in guiding the adaptive solution based on the thermal gradient., was investigated.

The results have shown that the RNIEP method is more accurate than the RNISP method in estimating the error, and the RNIEP method has provided a better strategy for improving the network using the t-r refinement method, and it has performed better than the RNISP method in obtaining more accurate, reliable solutions. And it can be used as an effective and simple solution to improve the stress field.

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