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Workspace of 3-RRR parallel robot using the combination of interval analysis and refinement methods by considering the limitation of active joints movement

Payam Varshovi-Jaghargh[1], Mehdi Tale Masouleh[2], Mehdi Karimi[3], Fatemeh Pourkariman[4]

[1]Assist. Prof., Faculty of Engineering, Mech. Eng., Hamedan University of Technology, Hamedan, Iran
[2]Assoc. Prof., Human and Robot Interaction Lab., School of Electrical and Computer Eng., University of Tehran, Tehran, Iran
[3] Assist. Prof., Faculty of Engineering, Mech. Eng., Bu-Ali Sina University, Hamedan, Iran
[4] MSc, Faculty of Engineering, Mech. Eng., Bu-Ali Sina University, Hamedan, Iran

*Corresponding author: varshovi@hut.ac.ir

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Abstract

In this paper, two new algorithms are presented for calculating the workspace of parallel robots considering the limitation of active joints movement by combining interval analysis with two refinement methods. In proposed methods, the kinematic expressions of the chains are calculated based on the joint coordinates and geometrical constraints governing the kinematic chains of the parallel robot. In the following, two methods are presented for calculating the workspace of parallel robot by combining the concepts of interval analysis with two refinement algorithms, namely mean value form and slope form. The two presented algorithms and the interval analysis method without refinement operation are implemented on the 3-RRR planar parallel robot and the constant-orientation workspaces are obtained for different limitation of active joints movement. For the sake of comparison, the constant-orientation workspace is also obtained using the geometric method, and it is shown that if refinement is used in the interval analysis, the workspace of the parallel robot is entirely consistent with the results of the geometric method.

Keywords: Parallel robot; Workspace; Limitation of joints movement; Interval analysis; Refinement.

1. Introduction

A parallel robot is a closed-loop mechanism in which the moving platform is connected to the fixed platform by at least two serial kinematic chains [1]. The workspace determination plays an essential role in the design, construction and operation of parallel robots.

Most of the investigations in the field of parallel robot workspace have been done using geometric or numerical approaches. In recent years, some researchers have conducted studies on the workspace of parallel robots based on interval analysis. In most of them, the geometry of mechanism is considered as the constraint in the workspace analysis. But in none of them, the refinement operation has not been used in the interval calculations. Gouttefarde et al. [2] obtained the wrench-feasible workspace of parallel cable-driven robots using methods based on interval analysis. Kaloorazi et al. [3] presented an interval-based approach for determining the obstacle-free workspace of two types of parallel robots containing one prismatic actuated joint per kinematic chain, named 3-RPR planar parallel mechanism and 6-DOF Gough–Stewart platform. Jin et al. [4] proposed an interval-analysis-based algorithm for trajectory-reachable workspace of cable-suspended parallel robots. Ye et al. [5] used an interval discretization method to find the workspace of parallel robots.

The refinement operation is very essential in the accurate calculation of the parallel robot workspace. If the refinement operation is not used in the interval analysis, the interval obtained for each of the kinematic equations will have an excess width [6]. This factor causes the resulting workspace to be larger than the actual workspace of the parallel robot. According to this issue, in this paper, two systematic algorithms are presented for analyzing the workspace of parallel robot by using the combination of kinematic equations, limitation of joints movement, interval analysis and two refinement methods, named mean value form and slope form. The mentioned two algorithms are implemented on the 3-RRR planar parallel robot and the obtained results are compared with the results of geometric method.

2. The workspace of 3-RRR parallel robot using two proposed algorithms

The structure of the proposed algorithms is briefly described below for calculating the parallel robot workspace. The schematic representation and geometric parameters of the 3-RRR parallel robot are respectively shown in Fig. 1 and Table 1. The kinematic expressions of the first to third kinematic chains of the 3-RRR parallel robot are obtained as follows by using the coordinates of the joints, the constraints governing the kinematic chains of the parallel robot and the elimination method.

$$\begin{aligned} f_1 &= B_1C_1 \\ &= (x - l_1c_1)^2 + (y - l_1s_2)^2 - l_2^2; \end{aligned} \quad (1)$$

$$\begin{aligned} f_2 &= B_2C_2 \\ &= x^2 + y^2 + l_1^2 - l_2^2 + a^2 + b^2 \\ &+ 2bys_\theta - 2ax - 2(a - x)bc_\theta \\ &+ 2(a - x - bc_\theta)l_1c_2 \\ &- 2(bs_\theta + y)l_1s_2 \end{aligned} \quad (2)$$

$$\begin{aligned} f_3 &= B_3C_3 \\ &= x^2 + y^2 + l_1^2 - l_2^2 + a^2 - ax \\ &+ 1.25b^2 - 2yl_1s_3 + (a - 2x)l_1c_3 \\ &- (l_1c_3 + 2l_1s_3 + 0.5a - x - 2y)bc_\theta \\ &+ (2l_1c_3 - l_1s_3 + a - 2x + y)bs_\theta \\ &- (bc_\theta + 0.5bs_\theta - l_1s_3 + y)\sqrt{3}a \end{aligned} \quad (3)$$

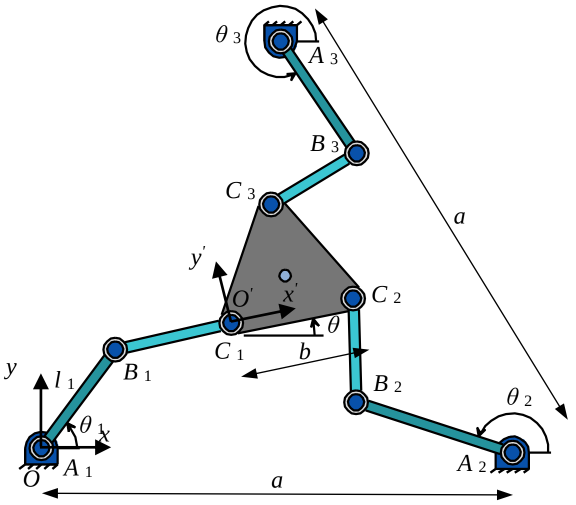

Figure 1. The schematic representation of the 3-RRR parallel robot.

The above expressions describe the position and orientation of the moving platform (x, y and θ) in terms of the variables of the active joints (θ_1, θ_2, θ_3) and form a system of nonlinear equations. The intervals of θ_1, θ_2, θ_3, x and y are given as input to the system of equations and the intervals of equations f_1 to f_3 are obtained by interval calculations and refinement operations. The intervals of x and y should be chosen so large that the workspace of the parallel robot is placed inside it. In the presented algorithm, an interval belongs to the workspace, if there is zero in all the obtained intervals of the f_i ($0 \in f_i$). In order to calculate the intervals of equations f_1 to f_3 more accurately, the refinement operation must be performed simultaneously. Due to the complexity of the system of kinematic equations, if the refinement operation is not used, the interval obtained for each of the equations will have excess width. In the algorithms presented in this paper, slope form and mean value form are used as two refinement operations to remove the excess width of the intervals of kinematic equation. If the zero belongs to all the intervals obtained for f_1 to f_3, the largest interval between x, y, θ_1, θ_2 or θ_3 is divided into two intervals. Then, each created interval is separately substituted into the system of equations, the intervals of f_i are recalculated and the condition of the existence of the solution is checked ($0 \in f_i$). The operation of dividing the intervals and checking the condition of existence of the solution should be repeated until the width of the intervals of the variables reach the desired accuracy (ε). The set of remaining intervals form the workspace of parallel robot. It is necessary to explain that the interval analysis is performed in INTLAB[1] varsion 11, which is the software package installed in MATLAB.

Table 1. The geometric parameters of the 3-RRR parallel robot.

a (cm)	b (cm)	l_1 (cm)	l_1 (cm)
120	40	30	30

3. Results

The constant orientation workspace of the 3-RRR parallel robot was obtained using three described methods, combination of interval analysis and mean value form, the combination of interval analysis and slope form and the general form of interval analysis, for the values of the geometric parameters shown in Table 1 and the different limitations of active joints movement. The results of three mentioned methods were compared with the geometric method. For example, the workspace of the 3-RRR parallel robot is shown in Fig. 2 for θ_1=[20°, 70°], θ_2=[110°, 160°], θ_3=[245°,295°], θ=0° and the accuracy value of 0.1172cm.

Comparison of the values in Table 2 and other obtained results shows that the workspace area of the 3-RRR parallel robot is very close to the actual workspace area when using the combination of interval analysis and refinement methods. But, if the refinement operation is not used, the area of the workspace is much larger than the actual workspace of 3-RRR parallel robot. These results clearly state the undeniable influence of the refinement operation in determining the workspace of the 3-RRR parallel robot. On the other hand, considering the time required to execute the algorithm, it is concluded that the combination of interval analysis with the mean value form takes less time compared to the combination of interval analysis with the slope form.

[1] Interval Laboratory

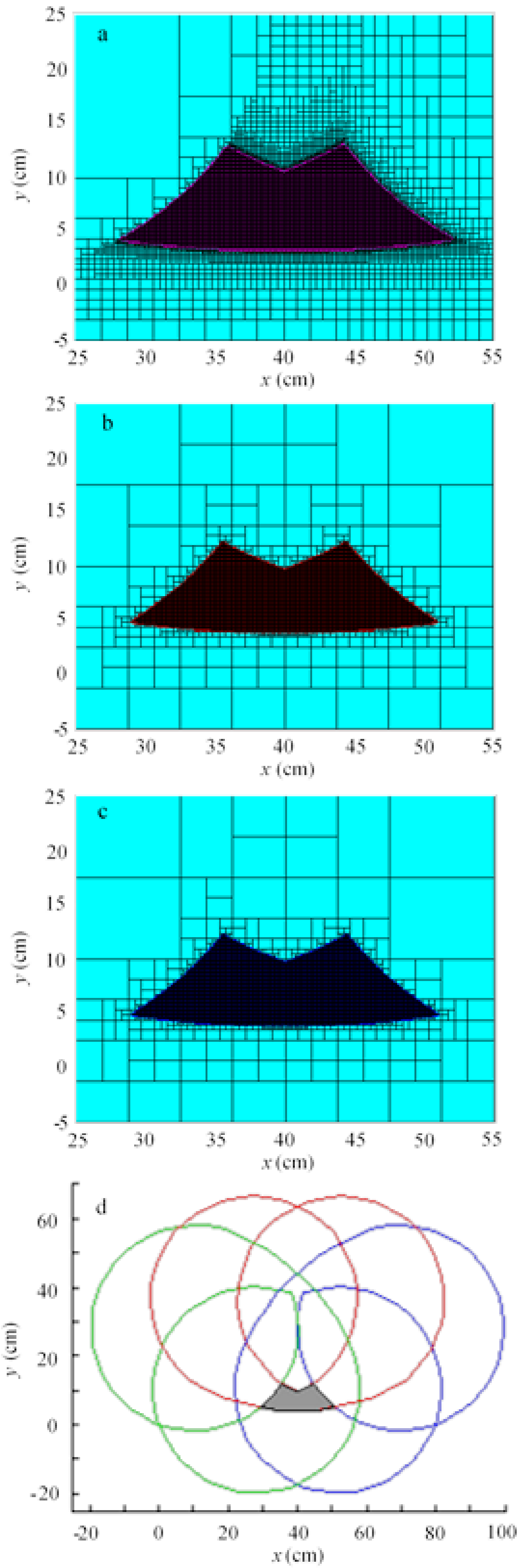

Figure 2. the workspace of 3-RRR parallel robot for θ_1=[20°, 70°], θ_2=[110°, 160°], θ_3=[245°,295°] using a) general form of interval analysis without refinement b) combination of interval analysis with mean value form c) combination of interval analysis with slope form d) geometric method.

Table 2. Workspace area and algorithm execution time for θ_1=[20°, 70°], θ_2=[110°, 160°], θ_3=[245°,295°]

Method	Area (cm^2)	Time (s)
General form of interval analysis without refinement	134.6	249390
Combination of interval analysis with mean value form	110.7	36038
Combination of interval analysis with slope form	110.8	8773
Geometric method	106.8	-

4. Conclusions

In this paper, two algorithms were presented to investigate the workspace of parallel robots by combining interval analysis and refinement operations. The kinematic equations of parallel robot, limitation of joints movement, interval analysis, mean value form and slope form were used for the proper analysis of the workspace. The results of this research showed that if the refinement operation is not used in the interval analysis, the calculated workspace is geometrically similar to the actual workspace of the parallel robot, but its area is significantly larger than that. The combination of interval analysis with one of the refinement operations, such as mean value form or slope form, reduces the excess width of intervals of the kinematic equations and the results are obtained similar to the actual workspace of the parallel robot both in terms of geometry and area. Moreover, it was observed that the use of the mean value form compared to the slope form spends less time in the presented algorithms based on interval analysis and is more suitable for the analysis of the parallel robot workspace.

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