Journal of Solid and Fluid Mechanics (JSFM), 13(2): 1-4, 2023

Shahrood University of Technology

Journal of Solid and Fluid Mechanics (JSFM)

Manufacturing Engineering of Iran

DOI: 10.22044/JSFM.2023.10853.3401

Nonlinear Vibrations of Functionally Graded Porous Micropipes Conveying Fluid Flow

Mohammad Ali Sabahi[1], Ali Reza Saidi[2, *]

[1] Ph.D. Student, Department of Mechanical Engineering, Shahid Bahonar University of Kerman, Kerman, Iran.
[2] Prof., Department of Mechanical Engineering, Shahid Bahonar University of Kerman, Kerman, Iran.
*Corresponding author: saidi@uk.ac.ir

Received: 05/22/2021 Revised: 04/17/2023 Accepted: 05/24/2023

Abstract

In this paper, an analytical solution for the nonlinear free vibrations of the functionally graded porous micropipes conveying fluid flow by using homotopy analysis method is presented. The equations of motion are obtained based on Euler-Bernoulli beam theory and modified couple stress theory with consideration of geometric nonlinearity. It is assumed that the micropipe is porous and the porosity distribution is in three forms; uniform, non-uniform symmetric, and non-uniform asymmetric distributions. The Hamilton principle is used to obtain the governing equations of motion. Also, the Galerkin method is used to convert partial differential equations to ordinary differential equations. Finally, by considering immoveable simply-supported boundary conditions and using the homotopy analysis method, the analytical solution for the governing equation is performed. The results obtained from this method has been verified by the Runge-Kutta numerical method which shows that the homotopy analysis method has good accuracy by considering two terms of the Taylor series expansion. The results showed that between the proposed porosity distribution schemes in the micropipe, the non-uniform asymmetric distribution pattern is the most suitable, because the microtube becomes unstable at a higher fluid velocity.

Keywords: Nonlinear Vibrations; Micropipe Conveying Fluid Flow; Functionally Graded Porous Materials; Modified Couple Stress Theory; Homotopy Analysis Method.

1. Introduction

The analysis of micropipes conveying fluid flow has attracted the attention of many researchers due to its many applications. Among the applications of fluid-conveyed micropipes, one can remark the utilization in medical science and industry. In medical science, micropipes are utilized to inject drugs into cancerous tissues. By applying this method, the amount of drug consumption is minimized and it is more efficient than traditional methods. Using micropipes conveying fluid in biosensors, heat transfer and semiconductors are the examples of micropipes applications in industry.

The mathematical modeling of linear vibrations and buckling of various pipes (from macro to nano scale) under different boundary conditions (e.g. clamped, free and hinged) made of functionally graded porous materials based on various theories have been carried out by numerous researchers [1-3]. Previous experimental studies showed that using classical theories of continuum mechanics is not appropriate for the dynamic modeling of micro- and nano-scale structures, and instead, the usage of modified couple stress theory is suggested [4].

The static condition and dynamic behavior of microbeams according to the non-classical beam model laying on elastic foundation were investigated by Şimşek [5]. Nonlinear free vibrations and self-excited of clamped micropipes conveying fluid considering gravitational effects were studied by Hu et al. [6]. Dehrouyeh-Semnani et al. [7] explored the nonlinear forced vibrations of micropipes subjected to harmonic external forces by applying Runge-Kutta method to solve partial differential equation. Babaei et al. [8] analyzed the nonlinear dynamic behavior of FG curved micropipes in thermal environment subjected to uniform lateral pressure.

In this paper, we explore an analytical solution for nonlinear vibrations of microtubes conveying fluid made of porous functionally graded materials based on homotopy analysis method. The equations of motion are based on Euler-Bernoulli beam model in conjunction with modified couple stress theory and considering geometrical nonlinearity. The governing equations of motion are obtained by applying Hamilton's principle and by utilizing Galerkin technique, the resulted equations are discretized. Finally, Homotopy analysis method is employed to solve the nonlinear differential equation.

2. FG Porous Microtube

Assume a micropipe with outer radius, r_o, inner radius, r_i, length, L, and mean radius, r. The velocity of fluid flow is Γ along the micropipe length in the x- direction. Also, it is assumed that the microtube is made of porous FG materials, in which the mechanical properties according to the various porosity distribution patterns are formulated as

$$\{E,G\}=\{E_1,G_1\}(1-e_0\alpha)$$
$$\rho=\rho_1\sqrt{1-e_m\alpha} \tag{1}$$

$$\{E,G\}=\{E_1,G_1\}\left(1-e_0\cos\left(\frac{\pi\check{r}}{t}\right)\right)$$
$$\rho=\rho_1\left(1-e_m\cos\left(\frac{\pi\check{r}}{t}\right)\right) \tag{2}$$

$$\{E,G\}=\{E_1,G_1\}\left(1-e_0\cos\left(\frac{\pi\check{r}}{2t}+\frac{\pi}{4}\right)\right)$$
$$\rho=\rho_1\left(1-e_m\cos\left(\frac{\pi\check{r}}{t}+\frac{\pi}{4}\right)\right) \tag{3}$$

where

$$e_0=1-\frac{E_2}{E_1},\qquad 0\le e_0<1$$
$$e_m=1-\frac{\rho_2}{\rho_1},\qquad 0\le e_m<1$$
$$e_m=1-\sqrt{1-e_0} \tag{4}$$
$$\check{r}=r-\frac{r_i-r_o}{2},\qquad r_i\le r\le r_o$$
$$\alpha=\frac{1}{e_0}-\frac{1}{e_0}\left(\frac{2}{\pi}\sqrt{1-e_0}-\frac{2}{\pi}+1\right)^2$$

while E, G, ρ, t, E_1, G_1 and ρ_1 are Young modulus, shear modulus, fluid density, wall thickness, maximum Young modulus, maximum shear modulus and maximum density, respectively.

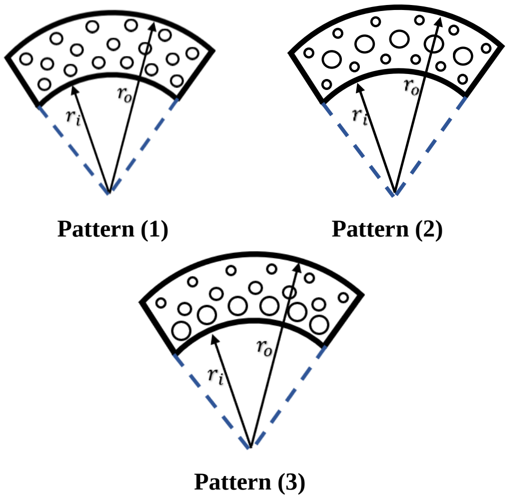

Figure 1. Various porosity distribution patterns

The governing equations of motion based on Euler-Bernoulli beam theory and modified couple stress theory with consideration of geometric nonlinearity, are as follows:

$$-\frac{\partial}{\partial x}\left\{\overline{EA}\left[\frac{\partial u}{\partial x}+\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2\right]\right\}+m\left(\frac{\partial^2 u}{\partial t^2}+2\Gamma\frac{\partial^2 u}{\partial x\partial t}+\Gamma^2\frac{\partial^2 u}{\partial x^2}\right)+M\frac{\partial^2 u}{\partial t^2}=0 \tag{5}$$

$$-\frac{\partial}{\partial x}\left\{\left[\overline{EA}\left(\frac{\partial u}{\partial x}+\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2\right)\right]\frac{\partial w}{\partial x}\right\}+m\left(\frac{\partial^2 w}{\partial t^2}+2\Gamma\frac{\partial^2 w}{\partial x\partial t}+\Gamma^2\frac{\partial^2 w}{\partial x^2}\right)+M\frac{\partial^2 w}{\partial t^2}+(\overline{EI}+\overline{GA}l^2)\frac{\partial^4 w}{\partial x^4}=0 \tag{6}$$

The above-mentioned relations are obtained through Hamilton's principle. where u, w, l, M and m are midplane displacement along x-direction, midplane displacement along y-direction, material length scale parameter, micropipe mass and fluid mass, respectively. also, the parameters are defined as

$$\overline{EI}=\int_0^{2\pi}\int_{r_i}^{r_o}E(r)r^2\sin^2(\theta)\,(rdrd\theta)$$
$$\overline{GA}=\int_0^{2\pi}\int_{r_i}^{r_o}G(r)\,(rdrd\theta) \tag{7}$$
$$\overline{EA}=\int_0^{2\pi}\int_{r_i}^{r_o}E(r)\,(rdrd\theta)$$

By considering the immovable hinged boundary conditions at both ends of the micropipe, one can write the governing equations as follows:

$$\overline{EA}\left(u+\frac{1}{2}\int\left(\frac{\partial w}{\partial x}\right)^2dx\right)=x\left[\frac{\overline{EA}}{2L}\int_0^L\left(\frac{\partial w}{\partial x}\right)^2dx\right] \tag{8}$$

$$-\left[\frac{\overline{EA}}{2L}\int_0^L\left(\frac{\partial w}{\partial x}\right)^2dx\right]\frac{\partial^2 w}{\partial x^2}+m\left(\frac{\partial^2 w}{\partial t^2}+2\Gamma\frac{\partial^2 w}{\partial x\partial t}+\Gamma^2\frac{\partial^2 w}{\partial x^2}\right)+M\frac{\partial^2 w}{\partial t^2}+(\overline{EI}+\overline{GA}l^2)\frac{\partial^4 w}{\partial x^4}=0 \tag{9}$$

3. Solution Procedure

Galerkin approach is employed to discrete the partial differential equation to ordinary differential equation as

$$\ddot{T}(t)+X_1\dot{T}(t)+X_2T(t)+X_3T^3(t)=0 \tag{10}$$

where

$$X_1=0$$
$$X_2=-\frac{\pi^2(\Gamma^2L^4m-(\overline{EI}+\overline{GA}l^2)L^2\pi^2)}{L^6(m+M)} \tag{11}$$
$$X_3=\frac{\overline{EA}\pi^4}{4L^4(m+M)}$$

Now by applying homotopy analysis method, we can obtain the nonlinear frequency and time response as follows:

$$\omega_{nl}=\frac{1}{2}\sqrt{3X_3a^2+4X_2} \tag{12}$$

$$T(t)=a\cos(\omega_{nl}t)+\frac{X_3a^3}{8{\omega_{nl}}^2}\{\cos(\omega_{nl}t)-\cos^3(\omega_{nl}t)\} \tag{13}$$

Where $T(t)$the time response of the center is point of micro pipe and ω_{nl} is the nonlinear frequency.

4. Validation Study

The results of this paper are verified with those reported by Dehrouyeh-Semnani et al. [7]. Figure 2 is depicted to show that the solution of homotopy analysis is accurate enough in which a good agreement between present study and data reported by Dehrouyeh-Semnani et al. [7] can be observed. It should be noted that in this figure, the non-dimensional frequency against the non-dimensional fluid velocity for various flexural rigidities are shown.

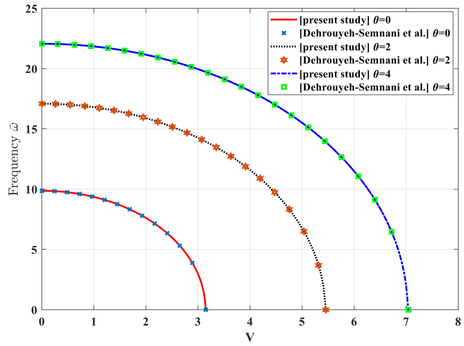

Figure 2. A comparison between present study and the results obtained by Dehrouyeh-Semnani et al. [7]

5. Discussion and Results

The numerical results of time history and nonlinear frequency are presented by using data of Table 1.

Table 1. Properties of micropipe and fluid

Name	Value	Symbol
inner radius	20μm	r_i
outer radius	30μm	r_o
length	15mm	L
material length parameter	15μm	l
maximum Young modulus of micropipe	200 GPa	E_1
maximum shear modulus of micropipe	75GPa	G_1
maximum density of micropipe	7850Kg/m^3	ρ_1
fluid density	1000Kg/m^3	ρ
initial amplitude	30μm	a
porosity	0.2	e_0

As it can be seen from figure 2 that by the increment of fluid velocity, the nonlinear frequency deceases until it becomes zero. When the nonlinear frequency becomes zero, the divergence instability happens. From Eq. (12), it can be easily concluded that the nonlinear frequency is always greater than linear natural frequency. Figure 3 is presented to show the effects of various porosity distribution patterns on the nonlinear frequency versus fluid velocity. As it can be seen, micropipe with distribution pattern 1 has the lowest frequency and micropipe with distribution pattern 3 has the highest. As it was pointed out that the micropipe may lose its stability by Divergence when the nonlinear frequency becomes zero; therefore, the critical fluid velocity in the micropipe with distribution pattern 3, has the highest value which means the non-uniform asymmetric distribution pattern is most suitable pattern.

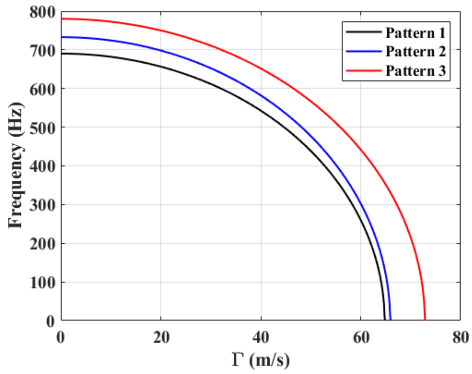

Figure 3. The effects of various porosity distribution patterns on the nonlinear frequency in terms of fluid velocity

Figure 4 is depicted to explore the effects of fluid density on the nonlinear time history. It can be realized that by the increment of fluid density, the time period of time history is increased. One can conclude that the nonlinear frequency deceases as the effective mass of micropipe system increases.

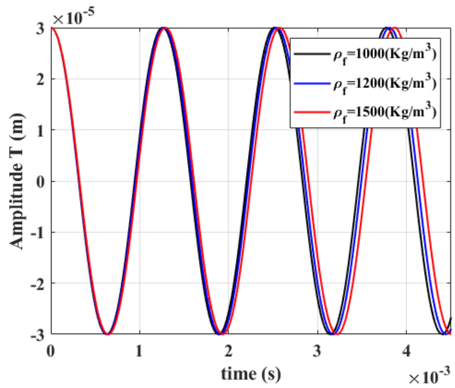

Figure 4. The effects of various fluid densities on the nonlinear time histories

6. Conclusions

In this work, we proceed to obtain the governing equations of motion by Hamilton's principle based on Euler-Bernoulli beam model and modified couple stress theory in conjunction with Von-Kármán nonlinearity relations. Three different porosity distribution patterns were considered for micropipe conveying fluid. The Galerkin method was applied to convert the partial differential equations to the ordinary differential equations. Finally, by considering immoveable simply-supported boundary conditions and using the homotopy analysis method, an analytical solution for the governing equation of motion was performed. The results showed that between the proposed porosity distribution schemes in the micropipe, the non-uniform asymmetric distribution pattern is the most suitable pattern. Also, by the increment of fluid density, the nonlinear frequency is decreased. Furthermore, the nonlinear frequency is always greater than linear natural frequency.

7. References

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