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Solving the inverse problem of identifying internal boundaries and estimating the mechanical properties of materials using optimization methods in materials with variable modulus of elasticity

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Abstract
One of the applications of optimization methods is solving the inverse problems of identifying internal boundaries, estimating the mechanical properties of materials and etc. In most of the articles, due to the simplicity of the relationships, cases where the material is a homogeneous body have been considered, But in industry, when two molten materials are combined, there is a possibility that the resulting material is non-homogeneous material. In this article, identifying the geometry of the irregular internal boundaries between three materials and estimating the mechanical properties is presented. The intermediate material has a variable modulus of elasticity. This problem has been studied by combining three methods of colonial competition algorithm, simplex and conjugated gradient method along with the numerical method of Boundary Elements Method. The effects of the type and hardness of the constituent materials of the non-homogeneous body and the effect of the geometry and position of the internal boundaries on the convergence have been investigated. The obtained results indicate the power and ability of the presented method to estimate unknown parameters.

Keywords: Boundary Elements Method (BEM), Imperialist Competitive Algorithm (ICA), Simplex Method, Optimization, Identification, Non- homogeneous body, variable modulus of elasticity.

1. Introduction

According to Figure 1, when material (1) is added to material (3) in the molten state, depending on the type of material, the resulting material (2) can be isotropic or non-isotropic, and the whole body will be Non-homogeneous altogether. In the direct problem, with the determination of boundary geometry, mechanical properties of materials and boundary conditions, unknown values of displacements and stress at internal and boundary points of the body are calculated. In the reverse problem, the mechanical properties of three materials, with internal boundary geometry are unknown, but the displacements on the outer surface of the body are measurable. In this paper, the formulation needed to solve the direct problem of calculating stresses and displacements on the external boundaries of Non- homogeneous body is presented [1-3]. Reference [4-6] is some of the work done in the field of identification problems. In this paper, identification of geometry of irregular internal boundaries between three materials and determining the internal structure of intermediate material in case of intermediate material has variable elasticity modulus, is investigated by combining three methods of Imperialist competitive algorithm, Simplex and conjugate gradient method along with Boundary Elements Method. The inverse problem is solved by minimizing the target function. In all the examples and cases solved in this paper, the objective function is defined as the sum of squares of difference between the measured displacement and the calculated displacement of the direct solution of the problem using the boundary element method. First, the Imperialist Competitive Algorithm (ICA) is used to determine unknown parameters. The obtained solution from the ICA is used as the initial guess for the Conjugate gradient method (CGM) and finally, to make the unknown values more precise, the solutions obtained from the CGM are used as the initial guess for the Simplex method. Also, the effect of the type and hardness of material and the effect of geometry and position of internal boundaries on convergence have been investigated.

2. Formulation of Boundary Element Method for Non- homogeneous Materials

The equilibrium equations for linear elastic problems are as follows:

$$\sigma_{ij,j}=0\longrightarrow\begin{cases}\dfrac{\partial\sigma_{11}}{\partial x_1}+\dfrac{\partial\sigma_{12}}{\partial x_2}=0\\\dfrac{\partial\sigma_{12}}{\partial x_1}+\dfrac{\partial\sigma_{22}}{\partial x_2}=0\end{cases}\qquad(1)$$

In the above relationship, $i,j=1,2$ along the axes x_2,x_1 and $\sigma_{11},\sigma_{12},\sigma_{22}$ are the components of tension.

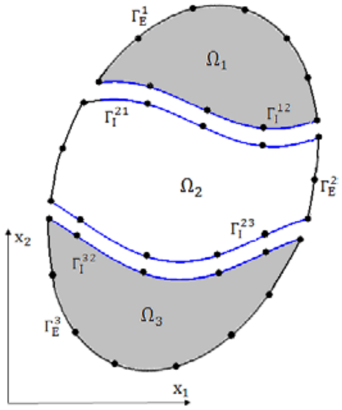

Figure 1. Non- homogeneous elastic body with three sub regions

With no lack of generality, assume that body forces vanish so that the equilibrium equations for the generic sub-region, are given by[3]:

$$[C_{ijhk}(x)u_{h.k}(x)]_{,j}=0\qquad(2)$$

Constitutive tensor $C_{ijhk}(x)$ is assumed to depend on the position vector, x by means of a scalar field expressed by the continuous function $\mu(x)$ and by a constant elastic tensor $\bar{c}_{ijhk}$.

$$c_{ijhk}(x)=\mu(x)\bar{c}_{ijhk}=\mu(x)[\frac{2\nu}{1-2\nu}\delta_{ij}\delta_{hk}+\delta_{ih}\delta_{jk}+\delta_{ik}\delta_{jh}]\qquad(3)$$

In Eq.3, $\mu(x),\nu,\delta_{ij}$ denotes shear modulus, Poisson's coefficient and Kronecker delta. Eq.2 can be rewritten as weighted residual vanishing equation:

$$\int_{\Omega}\left[c_{ijhk}(x)\,u_{h,k}(x)\right]_{,j}U_{li}(x,y)d\Omega=0\qquad(4)$$

Where the weighted function is chosen that satisfy the partial differential equation of the material described by Eq.3.

$$c_{ijhk}U_{li,jk}(x,y)+\delta_{lh}\delta(x-y)=0\qquad(5)$$

In Eq.5 $\delta(x-y)$ is the Dirac delta function at point y.The solution of Eq.5 is:

$$U_{ij}=-\frac{[3-4\nu)\delta_{ij}\ln(r)-r_{,i}r_{,j}]}{8\pi\mu(1-\nu)}\qquad(6)$$

Gauss's divergence theorem applied to Eq.4 yields:

$$\mu(y)u_i(y)=\int_{\Gamma}U_{ij}(x,y)t_j(x)d\Gamma-\int_{\Gamma}\mu(x)T_{ij}(x,y)u_j(x)d\Gamma+\int_{\Omega}\Sigma_{ijl}(x,y)\mu_{,l}(x)u_j(x)d\Omega\qquad(7)$$

In order to derive the necessary equations to calculate the stresses, it should be calculated. $\partial u_i/\partial y_h$ By deriving Eq.7 with respect to y_h and using Gauss theorem the following equation is obtained.

$$u_j(y)\mu_{,l}(y)\int_{\Gamma_\varepsilon}\Sigma_{ijl}(x,y)n_h(x)d\Gamma=J_{ijhk}^{\varepsilon}\mu_{,l}(y)u_j(y)\qquad(8)$$

The free term coefficients:

$$J_{ihjk}^{\varepsilon}=\frac{[(6-8\nu)\delta_{ij}-\delta_{ih}\delta_{jk}]}{16\pi(1-\nu)}\qquad(9)$$

If Eq.8 is considered on the borders, finally the following equation can be reached[3].

$$[U_\Gamma][u_\Gamma]-[T_\Gamma\quad\Sigma_\Gamma][t_\Gamma]=0\qquad(10)$$

In the above equation, u_Γ , t_Γ these are tractions and displacements on the boundaries and internal points.

3. optimization method

3.1. Imperialist Competitive Algorithm

Imperialist Competitive Algorithm (ICA) is a novel global search heuristic, that is inspired by the socio-political competition. Like other evolutionary methods, ICA starts with an initial population. Population individuals called countries are divided into two types: colonies and imperialists that all together form some empires. Some of the best countries in the population are selected to be the imperialists and the rest form the colonies of these imperialists. Imperialistic competition among these empires forms the basis of ICA. During this competition, weak empires collapse and powerful ones, taking possession of their colonies. Imperialistic competition hopefully converges to a state in which there is only one empire and its colonies are in the same position and have the same cost as the imperialist. After dividing all colonies among imperialists and creating the initial empires, these colonies start moving toward their relevant imperialist country. The colony moves toward the imperialist by x units. In this movement, θ and x are random numbers with uniform distribution. Further information about ICA could be found in [7].

Simplex method and Conjugate gradient methods are also used to solve the inverse problem in this paper, information about these methods could be found in [8].

4. The direct problem of calculating the stresses and displacements on the external boundaries of the multi- regions body

In this part, the method of calculating stresses and displacements on the external boundaries of a multi-region body, consisting of two homogeneous and one Non- homogeneous material, is examined. According to figure 1, the body is composed of three different materials, in such a way that material 1 and material 3 are homogeneous and the middle material is a Non-homogeneous material, in this case the domain is divided into three sub-domains, each of which has its own Poisson's ratio and modulus of elasticity and are separated by two internal boundaries. By applying the equations for each subdomain separately, we can write:

$$\begin{aligned}
&\begin{bmatrix} H_E^1 & H_I^{12} \end{bmatrix} \begin{Bmatrix} u_E^1 \\ u_I^{12} \end{Bmatrix} = \begin{bmatrix} G_E^1 & G_I^1 \end{bmatrix} \begin{Bmatrix} t_E^1 \\ t_I^{12} \end{Bmatrix} \\
&\begin{bmatrix} H_E^2 & H_I^{21} & H_I^{23} \end{bmatrix} \begin{Bmatrix} u_E^2 \\ u_I^{21} \\ u_I^{23} \end{Bmatrix} = \begin{bmatrix} G_E^2 & G_I^2 \end{bmatrix} \begin{Bmatrix} t_E^2 \\ t_I^{21} \\ t_I^{23} \end{Bmatrix} \\
&\begin{bmatrix} H_E^3 & H_I^{32} \end{bmatrix} \begin{Bmatrix} u_E^3 \\ u_I^{32} \end{Bmatrix} = \begin{bmatrix} G_E^3 & G_I^3 \end{bmatrix} \begin{Bmatrix} t_E^3 \\ t_I^{32} \end{Bmatrix}
\end{aligned} \qquad (11)$$

t_E^1, u_E^1 : The values of displacements and tractions in the nodes of the external border Γ_E^1 in the direction x_2, x_1 and so on. By applying the boundary conditions, the equations (11) can be combined and simplified as follows:

$$[H*] \begin{Bmatrix} u_E^1 \\ u_I^{12} \\ u_E^2 \\ u_I^{23} \\ u_E^3 \end{Bmatrix} = [G*] \begin{Bmatrix} t_E^1 \\ t_I^{12} \\ t_E^2 \\ t_I^{23} \\ t_E^3 \end{Bmatrix} \qquad (12)$$

In this equation, all the unknown values of displacement and traction are brought to the right and all the known values of displacement and traction on the boundaries are brought to the left, which is as follows:

$$[A]\{x\} = [B] \qquad (13)$$

The matrix of unknowns $\{x\}$ includes the unknown values of displacements and tractions on the external boundaries and all displacements and tractions on the internal boundarirs.

5. Results and Discussion

In the case of irregular internal boundaries, Young's modulus of middle material is assumed exponentially according to Figure 2 and is considered as follows:

$$E_2(y) = E_3 e^{\beta(y-y_{min})} \qquad (14)$$

$$\beta = \frac{1}{y_{max} - y_{min}} \ln\left(\frac{E_1}{E_3}\right)$$

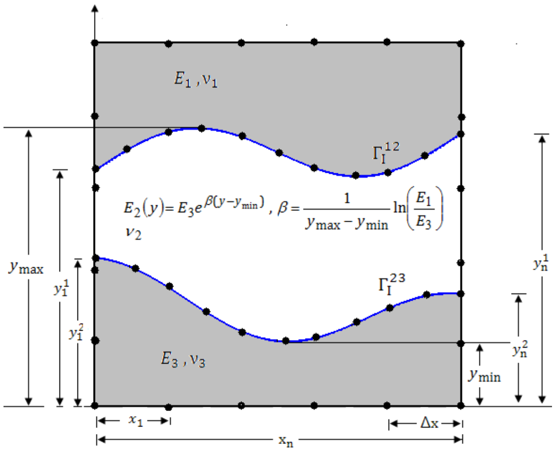

Figure 2. Non- homogeneous elastic body with three sub regions

To solve a problem in order to estimate internal boundaries and determine the internal structure of intermediate material, a Non- homogeneous body with irregular internal boundaries is considered according to Figure 2. The unknown parameters need to be estimated are 21 and are considered as follows:

$$[\Gamma] = [\Gamma^1 \quad \Gamma^2 \quad \Gamma^3] = [y_1^1, y_2^1, \dots, y_{10}^1, y_1^2, y_2^2, \dots, y_{10}^2, \nu_2] \qquad (15)$$

By identifying the internal boundaries, the value of y_{max} and y_{min} are calculated and by the placement in Eq.14, the internal structure of the intermediate matter is also estimated. The internal structure of the middle material is considered as follows:

$$E_2(y) = 210 e^{\beta(y-0.15)}$$

$$\beta = \frac{1}{0.8236 - 0.15} \ln\left(\frac{380}{210}\right) \qquad (16)$$

$$\nu_2 = 0.245$$

In order to solve the inverse problem, to identify internal boundaries and estimate the internal structure of the middle material, the Imperialist competitive algorithm is used. The Imperialist competitive algorithm, according to Figure 3, estimates the first and last nodes on each of the internal boundaries (four unknown parameters), estimating these two boundaries as a straight line. By estimating the initial guess by ICA, the continuation of the work is assigned by the Conjugate gradient method. To make the identified boundaries more precise.

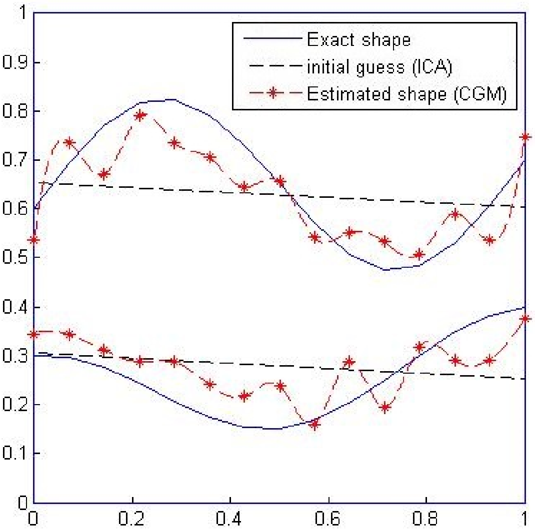

Figure 3. Identifying the boundaries of Non-homogeneous body with ICA and CGM

The answer obtained by CGM method is used as the initial guess for the Simplex method, and finally the estimated boundaries are obtained as shown in Figure 4.

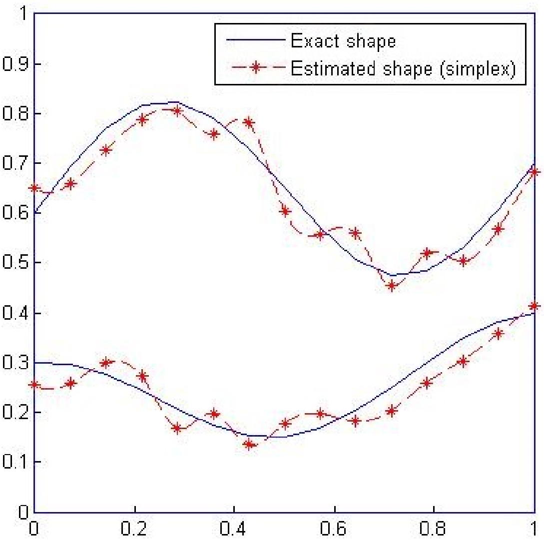

Figure 4. Identifying the boundaries of Non-homogeneous body with Simplex Method

6. Conclusions

In this article, identifying the geometry of regular and irregular internal boundaries between three materials, estimating the mechanical properties of materials and determining the internal structure of intermediate material were investigated. This was done by combining three methods of Imperialist competitive algorithm, as a general optimization method, to obtain the best initial guess, the Conjugate gradient method and the Simplex method as the local optimization method. Based on the obtained results, it can be said that the ICA can be effective in estimating an optimal initial guess for starting local algorithms. Simplex method is used as a method that does not require derivatives to identify the boundaries. The effect of this method is especially significant in the accuracy of the detection of internal boundaries and mechanical properties of intermediate material. It was observed that when the boundaries are near the external boundaries of the body, and when the difference in material properties of the sub regions decreases, the estimation is very good. One of the main applications of the proposed method in this study is its applications in the casting industry. One of the advantages of the proposed method is that the implementation of a test such as simple tensile testing is very cheap compared to other detection and identification methods such as laser or ultrasonic beams and therefore has economic justification in many cases.

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