

Identification of a Generalized Prandtl-Ishlinskii Model for a Micro Positioning System Actuated by Shape Memory Alloys Actuator

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Abstract

The application of Shape Memory Alloy (SMA) materials as an actuator has significantly increased in recent years. However, the application of SMA actuators is restricted due to their slow response and nonlinear complex characteristics including asymmetric hysteresis and saturated behaviors. Several methods have been introduced to model these actuators. One of the most powerful and well-known structures for modeling systems with hysteresis behavior is the Generalized Prandtl-Ishlinskii (GPI) model, which is widely used to control these actuators due to its analytical inverse. The current research investigates the increase in response speed of a shape-memory alloy micro-positioning platform using a two-way structure. Based on the obtained experimental results, an experimental-based model using the GPI model was identified. Using the inverse of this identified model and implementing it on the system, the nonlinear characteristics of the actuator is canceled and an equivalent linear dynamic system is considered for controller design. The results showed that the GPI model properly described the nonlinear behavior of the system despite the complexity caused by the interaction of two mutual actuators.

Keywords: System Identification; Shape Memory Alloy Actuator; Micro-positioning Stage; Generalized Prandtl-Ishlinskii Model; Smart Materials.

1. Introduction

Positioning systems are critical components of many mechanical engineering systems. Among these systems, micro-positioning stages are of particular importance due to their high accuracy and high sensitivity to the uncertainties. In recent years, researchers have shown increasing interest in micro-positioning stages with flexure hinge joints because of their ability to provide uniform, repeatable, frictionless and zero backlash movement. Although Piezoelectric materials are commonly used as the stimulating member in micro-positioning stages [1-3], their displacement range is limited and therefore achieving a reasonable range is necessary to increase the dimensions of the platform or to use magnification mechanisms. Unlike Piezoelectric materials, SMA actuators have high deformation capacity and can overcome this limitation. Due to low response speed and the existence of some complex nonlinear behaviors including hysteresis, the practical applications of Piezoelectric materials are limited. Consequently, research using these materials as actuators for micro-positioning stages has been limited [4, 5].

In this study, the authors discuss the system

identification of an innovative micro-positioning stage by optimizing the coefficients of the GPI model to extract its nonlinear hysteresis properties. The reasons for selecting and designing this platform are briefly described in section 2, and then the GPI model and its inverse are defined in section 3. The details of the test platform are introduced in section 4, and section 5 presents the results of the tests and the identified model for this system. The results, advantages, challenges, and limitations of this method are summarized.

2. Micro-Positioning Stage Design

Until now, most SMA based micro-positioning stages have a spring-back structure in which an actuator is employed for forward movement and the reverse movement is provided by a spring. In these systems, the stage movement in the opposite direction is achieved through various means, including cooling of the actuator, pushing by a spring-back, using external force in the opposite direction, or a combination of these methods [6, 7]. To achieve a suitable displacement range as well as to increase the speed of the stage movement, an innovative design has been proposed in

this study (as shown in Figure 1). The lateral deflection of the SMA beam is the source of movement. In this design, the stage returns to its original position more quickly by actuating the other actuator.

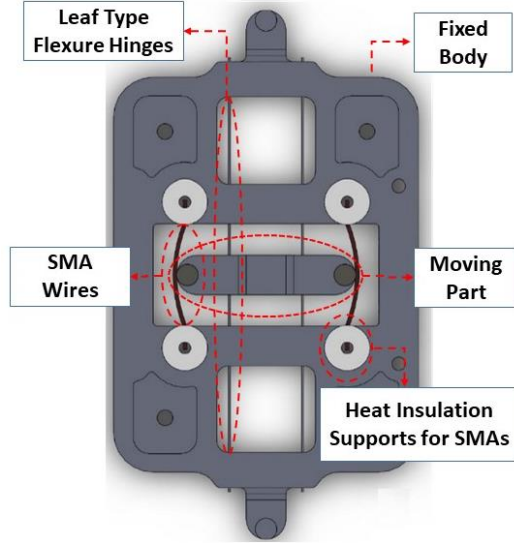


Figure 1. Designed Micro-positioning stage

3. An Overview on Generalized Prandtl-Ishlinskii model and Its Inverse

SMA actuators exhibit various non-linear properties, including hysteresis, during the cooling and heating process. One of the well-established models for modeling these nonlinear behaviors of smart materials is the GPI model. A detailed description of the extraction of this model and its inversion is available in [8], and only its results are utilized in this research. According to this method, the output of the GPI model Φ for an input function $v(t)$ is expressed as follows:

$$\Phi[v](t) = \int_0^R p(r) S_r[v](t) dr \quad (1)$$

In the above equation, $p(r)$ represents the density function, S_r is the generalized play operator, and r denotes the threshold values. For any input signal $v(t) \in C_m[0, T]$, where $0 = t_0 < t_1 < \dots < t_N = T$, the output of generalized play operator for $t_i < t \leq t_{i+1}$ and $i = 0, 1, \dots, N - 1$ is defined as:

$$S_r[v](0) = s(v(0), 0) = z(0) \\ S_r[v](t) = s(v(t), S_r[v](t_i)) = z(t) \quad (2)$$

$$s(v, z) = \max(\gamma_l(v) - r, \min(\gamma_r(v) + r, z))$$

where γ_l and γ_r are envelope functions and $\gamma_l \leq \gamma_r$. These envelope functions, density function, play operator and threshold values could be considered as follows:

$$\gamma_l(v(t)) = a_1 \tanh(a_2 v(t) + a_3) + a_4 \quad (4)$$

$$\gamma_r(v(t)) = a_5 \tanh(a_6 v(t) + a_7) + a_8$$

and:

$$r_i = a_9 i, a_9 > 0 \quad (5)$$

$$p(r_i) = a_{10} e^{-a_{11} r_i}, a_{10} > 0$$

These functions include 11 unknown parameters that must be identified through an optimization process. Once the GPI model is defined, its inverse can be derived analytically by performing some mathematical operations. The inverse of the model can be obtained as follows:

$$\Phi^{-1}[y](t) = \begin{cases} \gamma_l^{-1} \circ \sum_{j=1}^n \hat{p}(\hat{r}_j) F_{\hat{r}_j^+}[y](t) \text{ for } \dot{v}(t) \geq 0 \\ \gamma_r^{-1} \circ \sum_{j=1}^n \hat{p}(\hat{r}_j) F_{\hat{r}_j^-}[y](t) \text{ for } \dot{v}(t) \leq 0 \end{cases} \quad (6)$$

where:

$$\hat{r}_j = \sum_{i=0}^j p_i (r_j - r_i) \quad (7)$$

and:

$$\hat{p}_0 = \frac{1}{p_0} \quad (8)$$

$$\hat{p}_j = -\frac{p_j}{(p_0 + \sum_{i=1}^j p_i)(p_0 + \sum_{i=1}^{j-1} p_i)}$$

4. Experimental Setup

To evaluate the performance of the micro-positioning stage and investigate its nonlinear behavior, a positioning stage based on the design introduced in section 2 was constructed, and the necessary equipment for testing was also obtained. The experiment setup consisted of a computer device with LabVIEW software to control the system, a programmable power supply, a data acquisition card, a relay to switch the direction of electric current between the actuators, two SMA wires with a diameter of 1 mm, and a displacement sensor.

5. Results and Discussion

To eliminate the nonlinear hysteresis characteristic of the micro-positioning system, it is necessary to identify the parameters of the GPI model and its inverse. This process involves using the equations introduced in section 3 and performing an optimization process based on the nonlinear least squares method. In the first step, an input signal in the form of $\text{Sin}(\frac{\pi}{125} t)$ with a decreasing amplitude from 5 to 3.6 amps was applied to the system, and the output was measured. The *lsqnonlin* command in MATLAB software was then used to implement the optimization process. The optimal coefficients a_1 to a_{11} were obtained as described in Table 1:

Table 1. Identified parameters of GPI model using experimental data

Parameters	Optimized Values
a_1	7.61667
a_2	1.11619
a_3	1.51672
a_4	31.42343
a_5	0.25761
a_6	19.47572
a_7	-30.93615
a_8	0.86628
a_9	2.03529
a_{10}	-1.31329
a_{11}	0.42705

The output measured by the displacement sensor and the output simulated by the model were plotted in Figure 2. The results indicate that the model has successfully predicted the main hysteresis loop as well as minor loops.

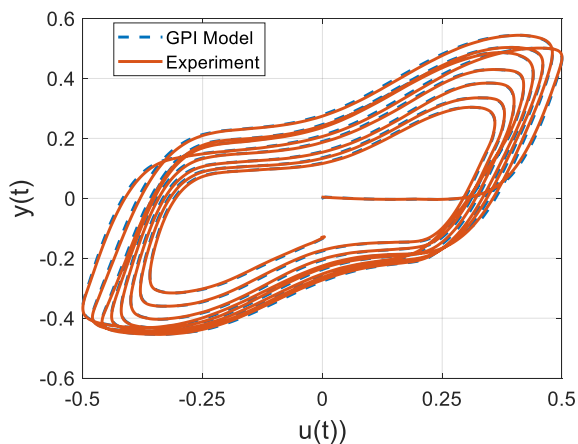


Figure 2. Response of the system and identified model to the input signal

To ensure the proper performance and validation of the model, a more complex input signal in the form of $u(t) = 2.5\cos(0.01\pi t) + 2.5\cos(0.02t)$ was applied to the system. The resulting output was measured and compared to the values predicted by the model. As shown in Figure 3, the comparison demonstrates that the model successfully predicts the system's nonlinear behavior for more complex input signals, providing further validation of the model's accuracy.

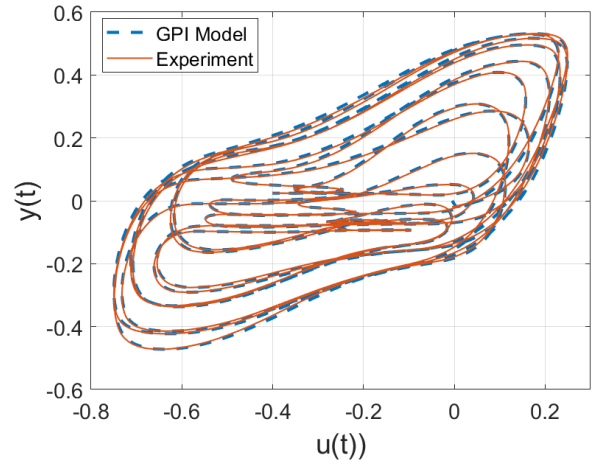


Figure 3. Response of the system and identified model to the validation input signal

In this experiment, it is clear that the identified model has successfully predicted the system response and the maximum error does not exceed 4%.

6. Conclusions

In the present research, a micro-positioning stage with two mutual actuators was introduced to increase the response speed of the system. Moreover, the identification and optimization of the GPI model, and implementation of its inverse to the system, have enabled the removal of the nonlinear hysteresis characteristic of the system, allowing for the use of standard linear control methods. However, it should be noted that the accuracy of the identified model may decrease when the complexity of the input signal increase and the system operates far from the identified point.

7. References

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