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Stability and dynamic analysis of Rayleigh functionally graded porous beams with longitudinal motion in hygro-thermo-magnetic environments

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Abstract

In this research, the influence of hygro-thermo-magnetic fields on the dynamics of axially moving functionally graded beams is investigated by considering various porosity models. Also, parametric studies are performed to clarify the effects of rotary inertia factor, the visco-Hetenyi substrate, material power index, follower force, and boundary conditions on vibration frequencies and instability threshold. The mechanical properties are graded transversely according to a power law. Different uniform and non-uniform porosity models are considered. The beam vibrates in variable moisture and humidity conditions, and is under an external longitudinal magnetic field. The dynamical equation is derived based on generalized Hamilton's principle and Rayleigh beam theory assumptions. With the aid of the Galerkin method, the eigenvalue problem is solved and frequency characteristics and instability boundaries are determined numerically. The axial velocity related to static instability is determined analytically. The results show that by increasing the porosity of the system with the first type of non-uniform porosity, the stability improves. Similar to hygro-thermal environments, the critical axial velocity decreases by increasing the power index. It has been proven that the stability decreases/increases by increasing the rotary inertia factor/magnetic field. The results could be useful for the design of axially moving inhomogeneous systems in complex environments.

Keywords: Functionally graded porous materials, vibration and stability, critical axial velocity, viscoelastic medium, longitudinal motion

1. Introduction

Axially moving systems have many applications in industry, including conveyor belts, robotic arms, and crane cables. Studies have shown that in the presence of axial movement, a displacement in the perpendicular direction of the motion may produce unwanted vibration. Therefore, dynamic modeling and vibration analysis of these widely applicable structures can play an important role in improving the performance of industrial structures [1]. Functionally graded materials are a category of composite materials whose properties change continuously in one or more directions. Due to the continuous gradient of mechanical properties of functionally graded materials, these materials provide better performance properties such as lower stress concentration and higher thermal resistance in comparison with conventional homogeneous and laminated materials. As a result, engineers have demonstrated great interest in utilizing these materials in axial moving structures [2]. One of the effective strategies of structural designers to improve the performance of industrial systems is the use of light porous materials in the construction of engineering structures. Recent studies have revealed that by characterizing graded materials with different distributions of internal pores, the dynamic behavior of systems can be excellently changed. Also, it is possible that during the process of production of graded materials, micro-voids/pores will be created in the structure. Therefore, functionally graded porous materials provide a unique potential for a wide range of applications in engineering sciences, especially moving structures. Therefore, investigation of the effects of pores on the vibration of functionally graded systems with axial motion is a mandatory engineering requirement [3]. However, limited studies have investigated the impact of functionally graded porous materials on axially moving systems. The performance of industrial equipment is highly dependent on its environmental conditions. For example, in the presence of thermal fields, thermal expansion and, as a result, thermal compressive stresses appear in the system, which ultimately leads to a change in the vibration behavior of the structure. As a result, it can be stated that the system vibration can be controlled by applying thermal fields. Therefore, predicting the mechanical behavior of axially moving industrial structures in complex environments is an engineering necessity [4].

Based on the authors' information, the vibration behavior and stability of axially moving beams constructed by transversely functionally graded porous materials in hygro-thermo-magnetic environments with different boundary conditions have not been studied analytically and numerically. Also, the impacts of various parameters such as viscoelastic substrate, follower force, and rotational inertia factor on the dynamics of axially moving systems have not been reported yet. In this article, the dynamic equation of a beam made of functionally graded materials with longitudinal movement is derived by considering different porosity distribution models. By solving the eigenvalue problem, the vibration frequencies of the system are calculated. Then, the stability of the system is examined by implementing numerical and analytical treatments. The results in different conditions are validated with the results of available reports in the technical literature. Finally, the effects of key parameters on the dynamics of the system are specified.

2. Problem Formulation

The strain energy variations of the system are given according to the following equation:

$$\delta U = \int_0^L \sigma_{\mathrm{x}} \delta \varepsilon_{\mathrm{x}} A \mathrm{d}x \tag{1}$$

The Kinetic energy is calculated by considering the effects of transverse and rotational displacement of the cross-section, according to the following equation:

$$T = \frac{1}{2}\int_0^L \left\{ m_0 \left(V^2 + \left(\frac{\partial w}{\partial t} + V \frac{\partial w}{\partial t} \right)^2 \right) + m_2 \left(\frac{\partial w}{\partial t\, \partial x} + V \frac{\partial^2 w}{\partial x^2} \right)^2 \right\} \mathrm{d}x \tag{2}$$

The external work of hygro-thermo-magnetic fields is calculated according to the following equation:

$$W_{\mathrm{e}} = \frac{1}{2}\int_0^L (N_{\mathrm{H}} + N_{\mathrm{T}} + N_{\mathrm{M}}) \left(\frac{\partial w}{\partial x} \right)^2 \mathrm{d}x \tag{3}$$

The conservative and non-conservative variations of follower force are described as follows:

$$\delta W_{\mathrm{V}}^{\mathrm{c}} = V \int_0^L (L - x) \frac{\partial w}{\partial x} \frac{\partial \delta w}{\partial x} \mathrm{d}x \tag{4}$$

$$\delta W_{\mathrm{V}}^{\mathrm{nc}} = -V \int_0^L \frac{\partial w}{\partial x} \delta w \, \mathrm{d}x \tag{5}$$

The variations in external work by the viscoelastic substrate are calculated as follows:

$$\delta W_{\mathrm{F}} = -\int_0^L N_{\mathrm{F}} \delta w \, \mathrm{d}x \tag{6}$$

The work done by the axial tensile force is obtained from the following equation:

$$W_{\mathrm{P}} = \frac{1}{2} P \int_0^L \left(\frac{\partial w}{\partial x} \right)^2 \mathrm{d}x \tag{7}$$

To derive the governing equation, Hamilton's principle is used according to the following relationship:

$$\delta \int_{t_1}^{t_2} (T + W_{\mathrm{F}} + W_{\mathrm{V}}^{\mathrm{c}} + W_{\mathrm{V}}^{\mathrm{nc}} + W_{\mathrm{P}} + W_{\mathrm{e}} - U) \mathrm{d}t = 0 \tag{8}$$

By applying Hamilton's principle, the governing dynamic equation of the motion of the system is obtained as follows:

$$\begin{aligned} &D_2 w'''' + m_0(\ddot{w} + 2U\dot{w}' + U^2 w'') - \\ &m_2(\ddot{w}'' + 2U\dot{w}''' + U^2 w'''') + \\ &(N_{\mathrm{H}} + N_{\mathrm{T}} + N_{\mathrm{M}} + V(L - x) - P) w'' \\ &+ k_1 w + k_2 w'''' + c(\dot{w} + u w') = 0 \end{aligned} \tag{9}$$

The Galerkin method is used to discretize the dynamic equation and obtain the reduced-order equation [5]. The discrete form of the dynamic equation of the system is expressed in the matrix form as follows:

$$\mathbf{M}\ddot{\mathbf{q}}(t) + \mathbf{C}\dot{\mathbf{q}}(t) + \mathbf{K}\mathbf{q}(t) = \mathbf{0} \tag{10}$$

By solving the eigenvalue problem, eigenvalues are obtained. The imaginary part of the eigenvalues are the vibration frequencies and the real part of the eigenvalues represent the damping in the system. When one of the natural frequencies of the beam and the real part of the eigenvalue has zero and positive values, respectively, static instability (divergence) occurs in the system. Also, when one of the natural frequencies and the real part of the eigenvalue have positive values, the system undergoes dynamic instability (flutter). In these cases, the corresponding axial velocities are called critical axial velocities [6].

3. Results and Discussion

The effects of foundation parameters on the vibration behavior are shown in Figure 1. Since the axial velocity has a reducing effect on the equivalent rigidity, with the increase of the velocity in the longitudinal direction, the vibration frequencies decrease. According to the figure, by considering the foundation, due to the increase in the effective rigidity, the vibration frequencies increase. In other words, compared with the case without a foundation, the system with a foundation has better stability, and static and dynamic instabilities occur at higher axial velocities. Therefore, it can be expected that by improving the mechanical properties of the substrate, the stability will be strengthened. Also, as can be seen, the hardening effect of the bending stiffness parameter on the dynamic behavior of the system is more noticeable compared to the effect of the Winkler elastic modulus parameter. Also, the hardening effect of the bending stiffness parameter is more tangible in higher vibration modes. In Figure 2, the fundamental vibration frequency and the real part of the eigenvalues branches are shown in terms of the axial velocity by considering the foundation damping. According to the figure, when considering the foundation damping, the critical axial velocity increases. Also, the frequency decreases in the damped foundation case. According to the figure, in the absence of foundation damping, the system loses its stability when the fundamental frequency becomes zero. While in the damped foundation state, the system can maintain its stability by reducing the fundamental frequency to zero and increasing the axial velocity. In addition, the real part of the eigenvalue branche loses their symmetry concerning the horizontal axis. So that they have a non-zero value for axial velocities lower than the critical value.

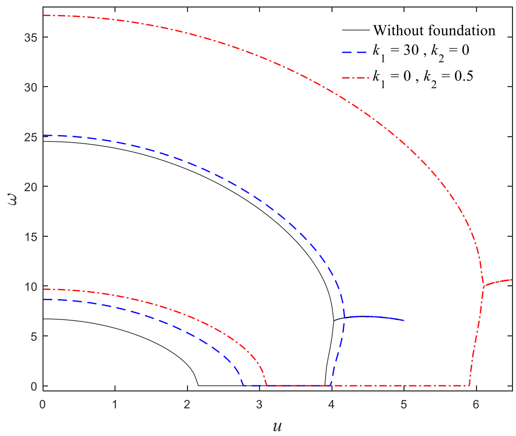
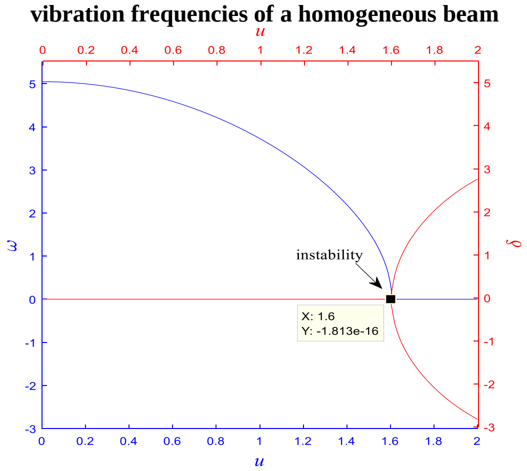
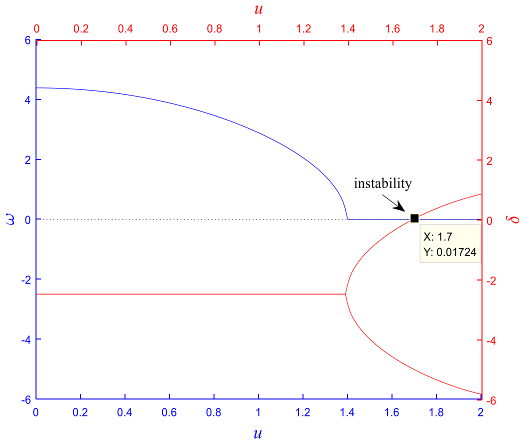


Figure 1. Effect of foundation parameters on vibration frequencies of a homogeneous beam



a



b

Figure 2. (a) Imaginary and (b) real parts of a homogeneous beam in terms of axial velocity

4. Conclusions

In this research, based on Rayleigh beam theory, the vibration and stability of axially moving functionally graded porous beams rested on a viscoelastic foundation with different boundary conditions under the effect of hygro-thermo-magnetic fields, axial and follower forces are analyzed. The dynamic equation is derived based on Hamilton's generalized principle. The reduced-order model equation is obtained with the aid of the Galerkin discretization method. The frequency characteristics and stability thresholds of the system are calculated with the help of numerical and analytical methods. The results of this research have been compared and validated with the reports in the literature. The results show that by increasing the viscosity parameter of the foundation, the stability of the system improves. Also, the bending stiffness parameter has a more stabilizing effect in comparison with the effect of the Winkler elastic modulus parameter. In addition, with the increase of foundation damping, the vibration frequency of the system 0decreases. Also, the outcomes revealed that the reducing effect of the follower force on the vibration frequencies can be reduced by increasing the bending stiffness parameter and the restraint of the beam supports.

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